HALF-BAKED MICROWORLDS AS EXPRESSIVE MEDIA FOR FOSTERING CREATIVE MATHEMATICAL THINKING

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Abstract

Creative thinking has been considered by policy makers as a fundamental aspect of human activity for the 21st century knowledge society and as an important lifelong skill that could promote innovation in the workplaces. Although research on creativity has been developing over the last few decades, the term "creativity" and its interpretation still hasn't been approached homogeneously. One of the processes that have been regarded as creativity indicators is the interplay between problem solving and problem posing. Constructionist media hold the potential for creative expression and original explorations, possibly entailing problem solving and posing processes, especially when students work collaboratively to construct meaningful for them artefacts. This paper describes the design of a web platform that entails a constructionist medium and two on-line shared workspaces. Empirical research with these tools attempts to enhance our understanding on they may support students in jointly figuring out how to fix a program for a 3D mathematical artifact and use it as a building block for creative constructions.

Keywords Creativity, constructionist learning, problem-solving, problem-posing, half-baked microworlds

1. Creativity and mathematical learning

Designing tools to support all students (and not just the gifted ones) in engaging in creative processes, has been a strand several researchers have turned to during the past few years. One of the main reasons for this is the emphasis policy makers have put on the need for novel and innovative ways of thinking, especially in times of economic and social crises. EU named 2009 as year of creativity and innovation and published a manifesto [1] explicitly stating that the one of goal for the following years would be to "make schools and universities places where students and teachers engage in creative thinking and learning by doing." ICT, considered to play an important role in the knowledge society of the 21st century and in the lifelong skills (essential for competiveness in the workplaces), is to be used as a tool for communication and creative expression as well.

Within the constructionist paradigm, researchers have been designing for years ICTs to be used by the students as media for creative expression and active explorations, allowing them to engage in the way in rich mathematical meaning making processes [2]. Such digital tools afford programming, dynamic manipulations and visualizations of complex situations driven by underlying mathematical rules - all usually offered through multiply connected representations. Although, these have been quite innovative technologies, designing tools and classroom pedagogies that can result in fostering creativity for all students has been proven a rather tall order. Creativity has been approached in an extensive number of ways, which led multiple researchers to suggest that the term "creativity" and the situations through which it may arise can't be specifically defined [3, 4]. To this end, the factors considered to be important when designing for creative mathematical thinking range from making sure that the students gain a good understanding of mathematics itself, to having deep experiences in doing mathematics, to establishing specific norms inside a mathematics classrooms [5, 6].

In this paper, however, we suggest designing tools that may offer opportunities for problem-posing and problem-posing activities, as means for fostering creative mathematical thinking [6]. We call those artefacts "half-baked microworlds" [7]. These are a special kind of microworld [8] as they are designed to be buggy or incomplete, aiming to challenge students to change them and make sense of the reasons for their unsatisfactory behavior. Considering the shaping of mathematical meaning making as a social activity, we design those artefacts to be questionable and improvable, with an intend to encourage students' participation in knowledge-building communities. By collaboratively tinkering the artefacts [9], we expect the students to debug the half-baked microworlds, reconstruct them and express their own ideas on how they should work.

Within such a community, the improved artefacts that the students make public and share on-line serve as "boundary objects" [10], i.e. artifacts obviating the need for any member of the community to understand what is already understood by another, but also becoming key in a mechanism through which the members of the community shape their collaborative activities and engage in joint meaning making processes [11].

In this research report paper, we describe instances in which such a community used on-line shared workspaces to pose and solve mathematical problems coming from its members' explorations with the "Twisted Rectangle" half-baked microworld. We discuss the affordances of the designed tools and how the participation in collectives and the joint tinkering of programmable artefacts created situations for creative mathematical thinking [12].

2. The Metafora System

The Metafora System [13] is a browser-based platform that offers two shared workspaces for individuals or groups of students to collaborate and communicate on-line as well as several domain tools with which they may address complex problematic situations. Among these, there are constructionist microworlds, such as a game-like physics simulation called Juggler, 3d math for creating and dynamically manipulating figures in 3d space and SuS-X, a kit for designing and playing game microworlds for environmental education.

The "Planning Tool" is the main shared workspace, opening up after logging in the Metafora System. It is designed to allow a group of students to structure a common plan of work (Figure 1) while working synchronously at distance. The students are expected to use it to jointly decide what the actions to take, in the different phases of their explorations, so as to address the problem in hand. For each action there is an available graphical card (with an extra text box to add any explanations), which the students drag from the inventory and place it inside the shared workspace. By connecting these cards, the students may organize their course of action within a span of at least 20 hours.

Apart from phases like "Explore", "Model", "Evaluate", there are also cards that depict the group's stances. For example, when "Evaluating" an artefact one of the students has prepared or "Discuss the Findings", the students may decide that they will need to be "Critical", as well as "Ethical" so as to make sure there is no prejudice. If special roles are decided to be allocated to certain group members, additional cards are available.

The Plan is available at any time and the students may go back and change it when they decide to follow a different direction in their explorations. For this reason, there is also a built-in chat available at all times and an awareness tool for the students to know who is around and what is doing.

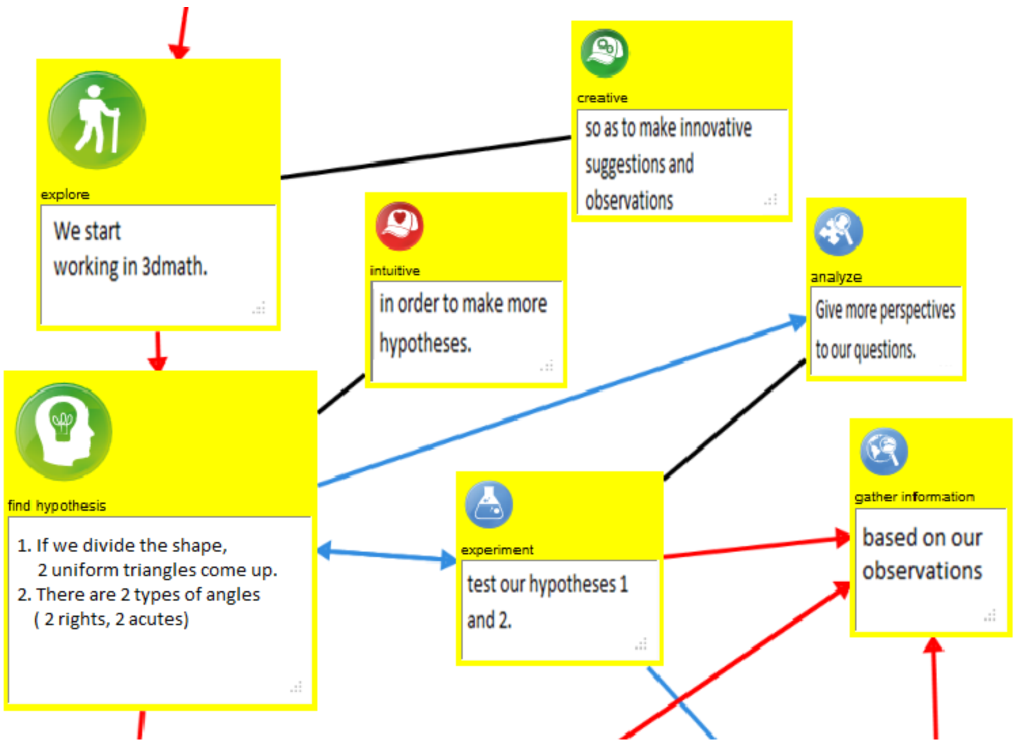

Figure 1: A part of the students' map – the yellow colour indicates that the students have tagged these Phases as finished ones

Negotiation and argumentation processes take place through "LASAD", a similar WYSIWIS environment (Figure 2). It allows the members of the group to synchronously discuss on-line. To do so, there is available a set of pre-defined text-boxes from which the students may select the one closer more suitable for the contribution they want to make. The title on the top of the box describes the type of the contribution they post, while drop-down lists allow them to further explain their idea. For example, they may add a "Microworld Action" text-box, to describe to others what they had been doing with their microworld or to suggest a new idea to be implemented by the group. From an additional list they may further indicate "What" is this microworld action (e.g. "Find the relation") and to "Which" part of the microworld it refers to (e.g. to the "Shape"). Linking these contributions together, the students create a kind of a map in which appears a graphical representation of their discussion.

Apart from communications, through LASAD and the Planning Tool, the students may also share parts or their whole work with the microworlds. Received microworlds, when uploaded by others, open in new tabs, allowing the students to view both their own and the others' versions of the microworld they are working with. In its early state, this functionality was performed manually, as the students were asked to copy-paste elements of their microworld they wished to make public within the community (e.g. parts of their Logo program and variable values for which the program should be run).

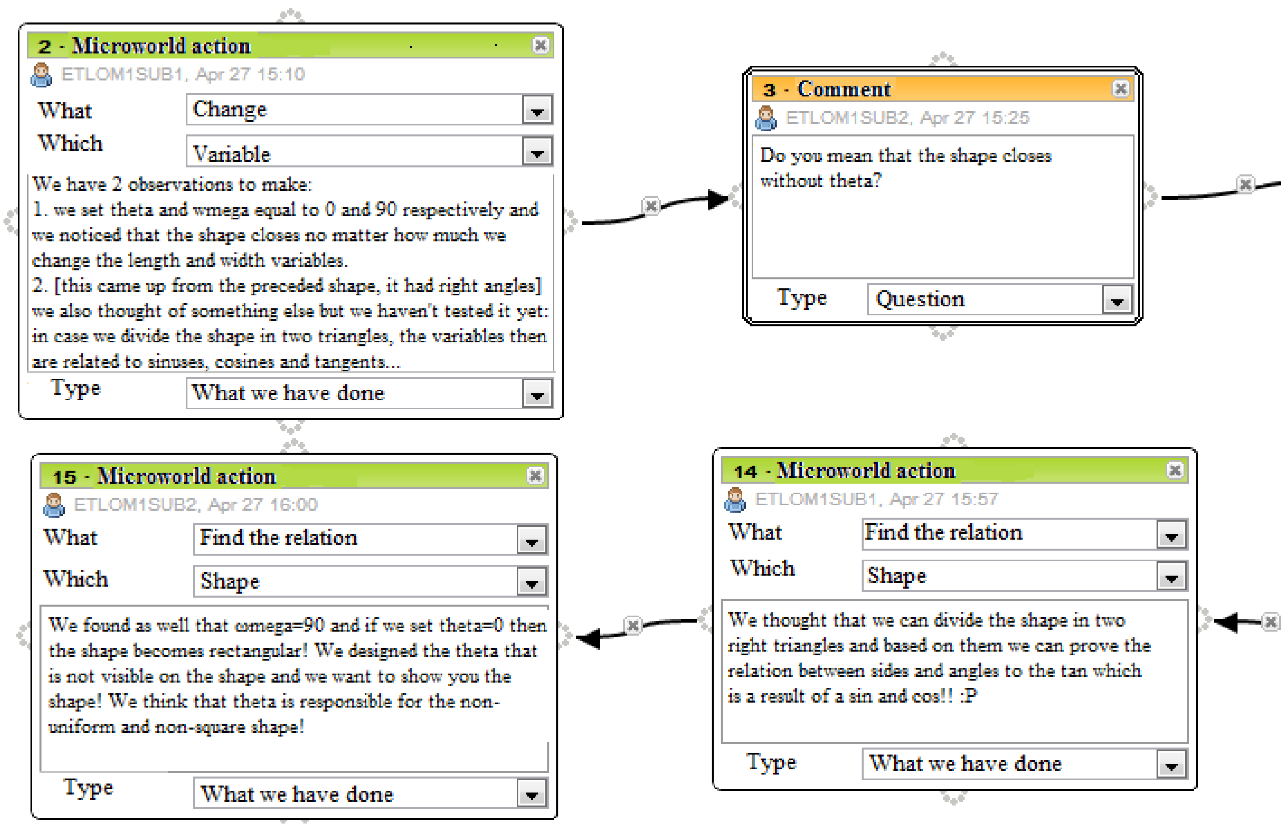

Figure 2: Students belonging to the same group exchange their observations as they implement each other's ideas on how to make a 3d shape close.

"3d Math" (http://etl.ppp.uoa.gr/malt) is a completely web-based authoring tool for mathematical learning. It is a 3d Logo programming Turtle Geometry tool, integrating affordances for dynamic manipulations of variable values and dynamic camera perusal [14, 15]. It builds on a previous standalone 2d version, called E-Slate "Turtleworlds". The 3d figures created within 3d Math, are results of programming commands that drive a turtle. The turtle's consecutive moves and turns in 3d space leave a linear trace behind, generating the figure. The dragging of a number-line variation tool affords dynamic "real-time" manipulations of the figure. This can be achieved as the variation tool controls the values of the variables in the program that initially created the figure. Continuous dynamic changes through this variation tool give an effect of a simulation.

Building and manipulating geometrical objects in 3d Math is not solely restricted to looking at the 3d world from static 2d views. A Camera Controller affords a dynamic change of the viewpoint that can be combined with a simultaneous zoom-in and zoom-out effect. Navigating around, inside and through their constructions the students may have the opportunity visualize 3d space as a whole and conceptualize mathematical notions related to stereometry. Thus, 3d Math can be used as an authoring tool for developing interesting 3d geometry half-baked microworlds, such as the "Twisted Rectangle" (Figure 3).

The Twisted Rectangle is a "buggy" 3d rectangle that has one segment twisting on a plane vertical to the one defined by the other three. It is designed to be an open-shape, aiming at challenging students to explore the mathematical relationships among the shape's lengths and angles as they attempt to close it.

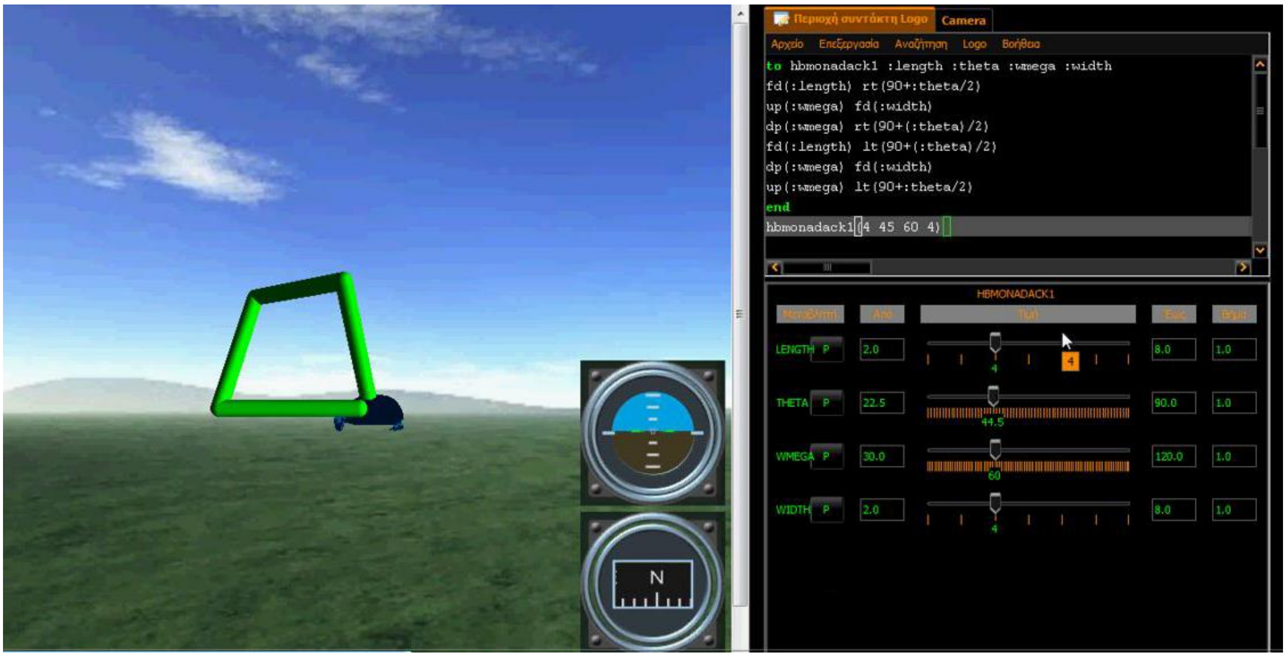

Figure 3: The "Twisted Rectangle" half-baked microworld

3. Context of Implementations and Methodology

The study described was implemented in a Lower Secondary Education School in Athens (1st Experimental Middle School) with ten 9th grade students (13 years old). Twenty-six school hours were dedicated to this research as a part of an after-class Math club in a span of about two months time (13 sessions).

The students worked together in two orchestrations: within a Subgroup and within a Group in which also participated other Subgroups (two to three Subgroups in total for each Group). In each Subgroup there were two and sometimes three students, working with the same 3d Math microworld in the same PC. Communication with the Subgroups sitting in other parts of the classroom took place on-line through LASAD and the Planning Tool. Just for the initial construction of the Plan, all Group members gathered together in front of a common PC. Alterations to the initial plan were discussed through LASAD.

The research approach was based on the idea of studying learning in authentic settings through design experiments [16]. Design experiments entail the engineering of tools and tasks in advance and the systematic studying of if and how these means support forms of learning in real classrooms. To this end, the researchers (also acting as the classroom teachers) chose not to intervene in the students' discussions and experimentations to give out specific instructions or to provide the "correct answer" to the students on how to address the challenge and proceed. Adopting a "participant observation" methodology, they preferred to pose meaningful -often intriguing-questions at certain time points, so as to encourage students to continue their explorations, elaborate more on their thoughts, share and discuss their ideas collaborating with in their Groups and Subgroups.

The data collected from cameras and a screen-capture software (HyperCam), recording the students' interactions with the Metafora System, were verbatim transcribed. The corpus of the data was completed by the students' LASAD maps, Plans, microworlds and the researchers' field notes. In analysing the data, we searched for instances in which the students, using the Metafora System, engaged in problem-posing and problem-solving processes. Our attempt was to identify how affordances of the system and especially of half-baked microworlds may support these processes, possibly leading to creative mathematical thinking.

3. Activities

Phase 1: The Twisted Rectangle microworld

The students were given the "Twisted Rectangle" half-baked microworld [9]. The "Twisted Rectangle" is a skewed quadrilateral, designed in a way that one of its vertexes seems to be "broken" as two of the corresponding segments were not connecting (Figure 3). Although it is obviously a faulty shape, the students were explicitly asked to work with the rest of their Subgroup members in order to "make the shape close". This entailed exploring the shape's geometrical properties and finding relationships among the shape's elements. To fix this "buggy behaviour", one of the variables in the Logo program needed to be expressed in terms of angles and segment lengths, also involving trigonometric functions.

As we didn't intend to provide any answers on how to achieve this goal, but ask students to discuss their ideas in an inter and intra Subgroup mode, we had already prepared a discussion space in LASAD in which the Subgroups could meet and share their findings as they explored this issue within the Twisted Rectangle microworld.

Phase 2: Constructions with the Twisted Rectangle as a building block

After completing the bug fix, the students in the Subgroups were asked to create their own constructions using the "Twisted Rectangle" as a building block. As members of a larger Group, they were expected to prepare a common Plan of work, discuss their ideas and share Logo programs or parts of their constructions with the other Subgroups through LASAD, so as to create meaningful for them complicated artefacts.

3. Results

1.2. Solving and posing new mathematical problems

After opening up the "Twisted Rectangle" microworld and playing with the simulation, the students came across right away with a question that was then explicitly asked by the researchers: "How can you make this shape become a closed one?". This was a quite generic problem and, at a first look, not a very mathematical one as it didn't give out any hints on what kind of mathematics would be needed or even if mathematics was needed at all.

To organise as a Group their ideas on how to address this problem, the students initially used the "Planning Tool". Placing and connecting together the one after the other "Phase" and "Activity" cards, they managed to translate the whole problem and its solving process into a diagram of actions (Figure 3). The free text space in the cards and the cards themselves were used to describe more eloquently the posed problem and the expected results that would signify that the problem has been solved.

Although this seemed to be clearly a problem solving technique (outlining and sequencing the generic steps to take), the students, though their Plan, also engaged in problem-posing processes. The "Find Hypotheses" card that they added (Figure 3), served as a venue for breaking the generic problem into simpler and more specific ones, which also appeared to be more manageable for them. The two new problems (subproblems) that the students came to pose in this card were: 1) "how to use only two variables to make the shape close" and 2) "how to relate to each other the three variables to make the shape close".

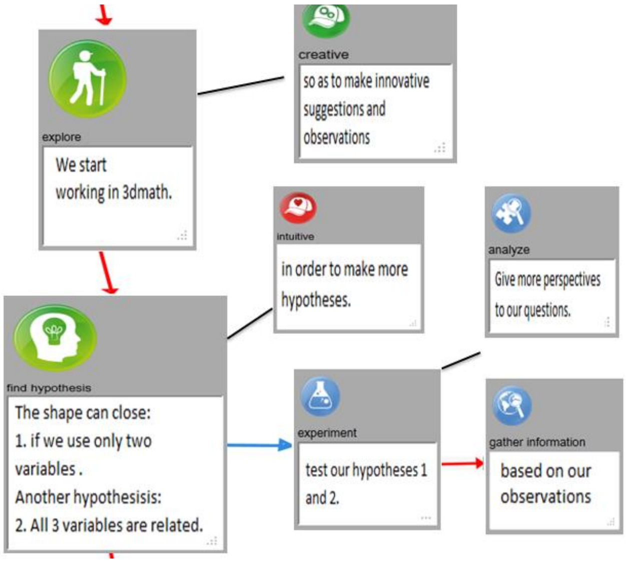

Figure 3: The Plan created by a Group of students

The first of the sub-problems seemed to be a special case of the generic one. Manipulating two of the three variables, by making the third one equal to zero, generated in 3d Math a simpler and more familiar problem. It was a problem that didn't correspond any longer to a 3d shape, but to a shape projected in a 2d plane. For 9th grade students, whose learning of trigonometry was limited and only for the 2d space, this new posed problem constituted a smaller "bite", fit to the mathematical knowledge they had been taught at school.

The second sub-problem was the original problem's *mathematical* translation. Although, the original question was asked in the context of "3d Math", it was not a clearly mathematical one. It didn't entail any mathematical terms (like "angles", "triangle sides" and "lengths"), but more-or-less, everyday language terms. In this "Find Hypotheses" card (Figure 3), the students restated the question, turning it into a more domain-specific problem that they could work with using the available resources. Looking for mathematical "relations among three variables", was a problem appropriate for 3d Math and its functionalities (programming using variables) and at the same time a problem one step more complicated than the first one.

Problem posing was a process that went on for students as they implemented their Plan and seemed to appear in-between problem-solving strategies. Experimenting with 3d Math, and through the "information they gathered" (Figure 3), after their "observations" didn't bring the desired results for some time, the students moved then to LASAD seeking for peer-to-peer evaluation of the outcomes of their explorations and joint decisions on how to continue as a group (Figure 2).

Again, this workspace, was used as a venue for reshaping a problem. This time, this was the initial subproblem. Instead of having just two variables (eliminating the third one by making it equal to zero), the students went one step further by suggesting another simplification: attributing a specific value to one more variable. However, this new special case with two out of the three variables referring to angles being equal to 0 or 90, was tested in 3d Math and rejected. Attempting to solve the problem through this second special case, led into having a new form of the shape which they called a "distorted shape".

After several dead-end approaches, one subgroup of students used some straws –for which they had asked a researcher in a previous session- to represent the Twisted Rectangle and a piece of paper to create a 2d plane beneath it (Figure 5).

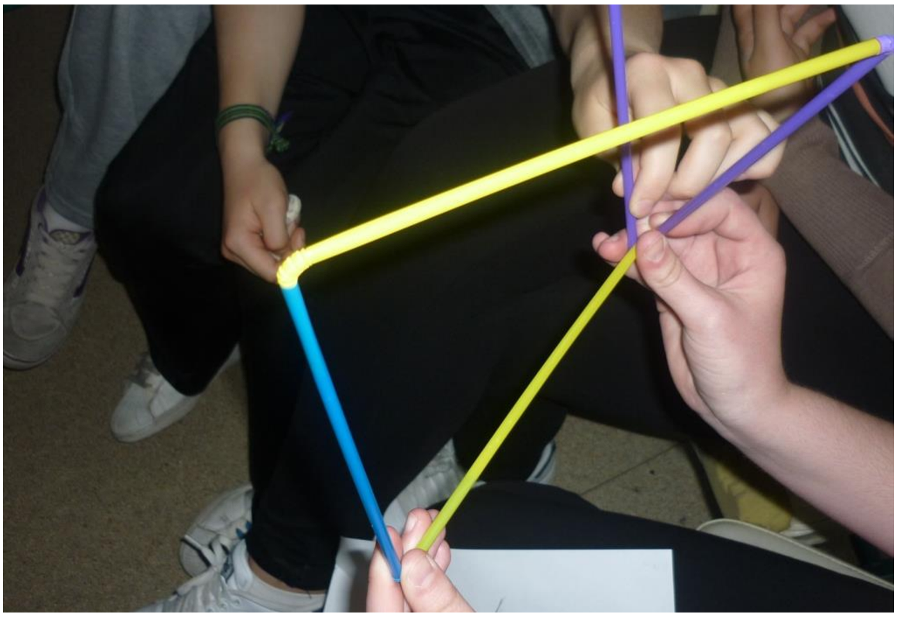

Figure 5: The same problems with other means of representation

Folding a part of the paper around the straws, they identified one more plane that they should take into consideration for closing the shape. This was a major breakthrough for the students, which resulted in creating projections of the shape in two separate planes. The problem of “how the three variables may relate to each other” was then turned into a simple and familiar for them trigonometry problem between two right triangles. To explain how the initial problem was cut down into pieces, they changed the “Find Hypotheses” card in their Plan indicating the solving strategy they followed (Figure 1).

1.2. Novel constructions

Having finished with the bug-fixing phase, the Subgroups used the experience they had gained during their experimentations to create novel constructions using the Twisted Rectangle as a building block. Having created a “participating in collectives” culture, the Subgroups of students shared their artefacts through LASAD in the form of Logo programs and discussed on how to combine them into a jointly constructed one (Figure 6).

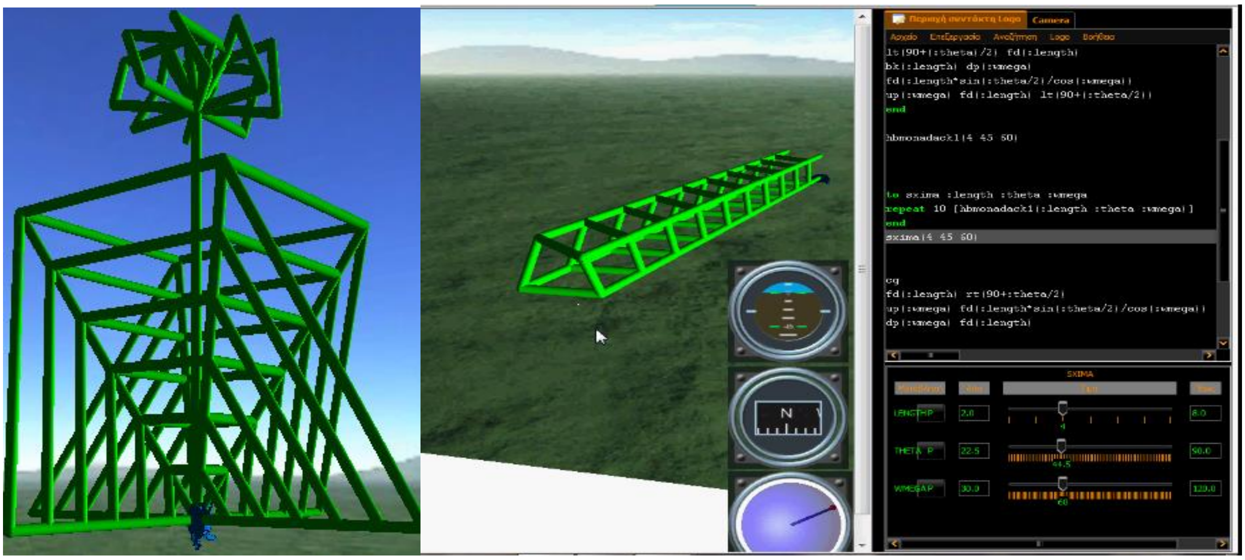

Figure 6: Constructions from two Groups – an artistic one and a more technical one

The integration of the artifacts initiated a new process in which the students engaged in two types of activities. The first one related to making sense of the artifact received and of the way it was constructed. The second one referred to how to combine it with their own so as to generate a more elaborated artefact. Again, these two types of activities corresponded to problem-solving (trying to understand what someone else created) and to problem-posing (sending back a new creation after having evaluated and/or integrated the received construction to their own for the others to solve the problem of understanding and integrating it).

5. Discussion

Understanding creativity as a term that has been coined through different approaches, we used in this paper the students’ problem solving and posing strategies to explain how they came to create novel and interesting artefacts as they worked with an web-platform. Our specific focus was on evaluating the design of the platform that included two on-line shared workspaces and a half-baked microworld with a buggy behaviour. The open-ended problem of "How to close the open shape" was thus chosen to call for planning of actions in advance, discussing in collectives and exploring mathematical ideas to fix the 3d shape.

As the students worked with the Platform's tools to address the problem, we signified several occasions in which they used interchangeably problem-solving and problem-posing strategies. Those were related to: cutting down the initial problem to smaller and more manageable pieces, translating it using terms close to the resources available for solving the problem, reshaping the problem creating special cases of the general problem, sharing the outcomes of the posed problems and assessing others' attempts, recognising situations in which the restated problem didn't lead to any results with regard to solving it, generating larger problems that combining the solutions of smaller ones.

The experience leaves us with a sense of needing to find out more about how to design affordances of tools that may create situations for problem-solving and problem-posing opportunities, aiming at studying if and how these may constitute a more solid indicator for the appearance of creativity.

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