3D TURTLE REPRESENTATION SYSTEM AND MENTAL ROTATION USING 3D TURTLE PERSPECTIVE

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Abstract

In this study, based on the fact that there is no appropriate cognitive tool to represent and analyze the shape of 3D cube stacks in Korean mathematics curriculum, the 3D turtle representation system was presented for the expression and communication of the 3D cube stack shape. Based on this 3D representation system, educational activities were designed and whether such activity could mediate the heuristic cognitive process of students for 3D cube stacks was investigated through MRT (Mental Rotation Test). Analysis of each item which is categorized into one of four types depending on the accuracy rate for each pre and post MRT revealed that in the pre evaluation, students solved problems by mentally rotating the objects through approximate sensual perception while the students solved problems using the 3D turtle perspective through embodied simulation in the post evaluation. The experimental results showed that the designed 3D educational activity altered the 3D cognitive process. Finally, the educational significance of the activity was assessed.

Keywords 3D object, 3D turtle representation system, turtle symbol system, turtle perspective, mental rotation test, 3D construction activity

1. Introduction

Recent advances in science and technology have brought realization of what we just dreamed of: for example, 3D films that enhance the illusion of the depth perception, and 3D printers that produce 3-dimmensional solid objects of virtually any shape from a digital model. Since we live in a 3D world, comprehending and communicating 3D figures are considered to be indispensable cognitive abilities for everyday life. The ability to recognize and control 3D figure is considered a critical ability in school curricula, and the Korean mathematics curriculum covers spatial cognition through 3D cube stacks, planar figures, polyhedrons, bodies of revolution, and vector geometry. In the 3D cube stacks chapter of the Elementary School 6th grade mathematics curriculum, the students learn to visualize and transform 3D cube stacks and communicate with each other about the shape of the 3D stacks. How do we recognize and remember 3D cube stack shapes? How do we visualize and mentally transform them? And, how can we communicate with others regarding spatial structure?

With its root in the Greek word for “to find” and “to discover”, heuristic refers to the simple and spontaneous reasoning based on the rule of thumb arithmetic, intuitive judgment, experience, or

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common sense to solve problems promptly encountered in uncertain and complicated situations. Research by psychologists Tversky and Kahneman made heuristic better known, which has become a core concept in understanding the decision making process under uncertainties. Even young children are able to instinctively judge a small or a large quantity, perform simple arithmetic, and differentiate shapes and colors. Such determinations are made through the intuitive and sense-depending heuristic approach. Going beyond the instinctive senses, however, humankind has invented and used symbol systems such as languages and number systems in order to recognize subjects more accurately. Such representation systems not only became the cognitive tools for humankind to make thinking more elaborate and advanced, but also contributed in making communication with others more efficient and convenient, which lead to the accumulation of human knowledge.

The process of recognizing a 3D object shape and determining its structure is also done heuristically, that is, roughly depending on our senses since there is no accurate expressive tool for 3-dimensionality. We have got into difficulty from time to time when we tried to express or manipulate a 3D object. And moreover, we felt helpless when we had to communicate with others regarding such object. In this study, the 3D turtle representation system was introduced as a tool for 3D object expression and communication, and based on this system, a 3D construction activity was designed. In order to investigate whether such activity can mediate the heuristic cognitive process of determining the structure of 3D cube stacks, pre and post MRT (Mental Rotation Test) were conducted. Finally, the educational values of the 3D activity designed in this study were assessed.

2. 3D Turtle Representation System

LOGO has been widely utilized in the field of the mathematics education in the U.S. since 1980 as a cognitive tool for constructing geometrical figures. In Korea LOGO is also introduced in the mathematics textbooks of the elementary school curriculum. Cho et al., recognizing that many students found it difficult to use the existing LOGO system as is a programming language, converted the “action command” to the simpler “action symbol,” and designed the “turtle representation system” to compose a 3D object [1][2]. In other words, while in the original LOGO system, the turtle moves drawing a line, in the 3D turtle representation system, the turtle moves stacking 3D cube. As shown in Figure 1, the turtle representation system is basically composed of the 6 action symbols: *s* and *t* that refer to moving forward and backward respectively, *r* and *l* that means moving right or left, and *u* and *d* representing moving up and down. Three versions of this representation system were developed according to whether the turtle moves within an absolute coordinate system or relative coordinate system [3].

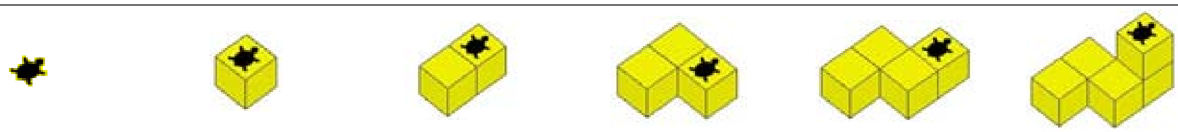
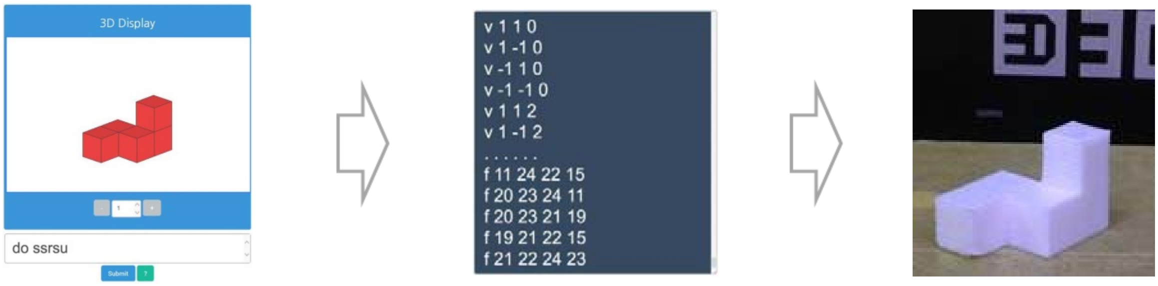


Figure 1. 3D representation using 3D turtle sysmbol system

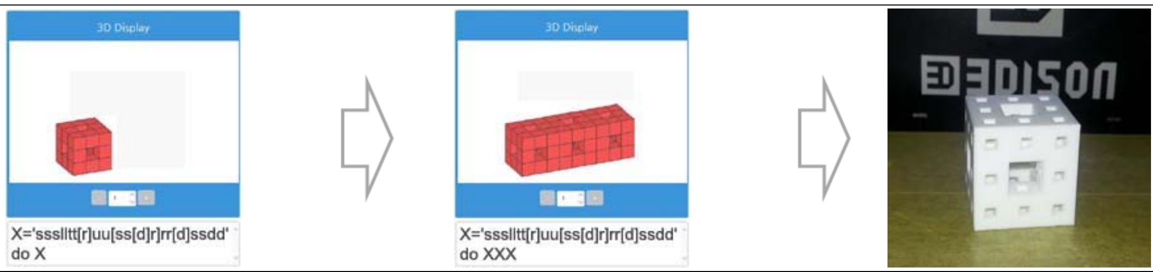
Using the 3D turtle representation system, we can construct a 3D object as we wish, which is shown on the screen both as a visual object and semiotic symbols. Cho et al. had designed a web-based creativity contest for students and evaluated its educational implication in [4]. In the designed creativity contest, the students were asked to design and construct their artifact using the 3D turtle representaion system. In their construction process, they not only created their own artifact but also formed a web-based interactive network to collaborate and share information among themselves. Also it was observed that the students were engaged in the cognitive thinking in order to utilize the semiotic 3D turtle symbols more efficiently. Recently, the advent of the 3D printer made it possible to produce a 3D solid object which is designed and expressed by the turtle representation system in virtual space. Figure 2 shows the process of how the turtle representation symbols are automatically translated into the object file so that the 3D printer can understand and perform the commands to pruduce the output. In a word, the turtle representation enables the creation of 3D objects in both the virtual and physical worlds through communication between the user and the machine.



turtle representation | object file | 3D printer output

Figure 2. 3D printing process using 3D turtle representation system

The 3D turtle representation is a kind of mathmatical representation system that can powerfully generate complicated 3D structure using mathmatical way of substitution and repetition. The Menger sponge has a fractal structure that can grow following the recursive pattern as shown in Figure 3. In the first picture, the turtle symbols that can create a 1st-step Menger sponge are substituted by a symbol X. The command to repeate the symbol X three times (i.e. do XXX) makes three many 1st-step Menger sponges successively. Like this, several times of substitutions and repeatitions of the turtle symbols will make a 2nd-step Menger sponge as shown in the third picture.



1st-step Menger sponge | 2nd-step Menger sponge

Figure 3. The process of making Menger sponge using 3D turtle symbol system[3]

3. Activity Design

The process of recognizing a 3D object and determining its structure is a heuristic approach based on intuition and experience. The purpose of this study was to investigate whether the intuitive cognitive process for 3D objects can be educated through a 3D construction activity which is designed based on the turtle representation system, and to evaluate its educational significance.

[3] In Figure 4, '[ ]' is a symbol to save a turtle's position and recall it. For example, when we command 'A[B]C', the turtle makes A, memorizes its position, makes B, then comes back to the remembered position, and makes C.

In the 2D Turtle Geometry, body-syntonicity is a critical concept [5] in that (a) to navigating the turtle by coordinating one’s body posture, physically or imaginary, with the turtle-vehicle of motion, and (b) to solving geometrical problems drawing upon ones embodied motional experiences. As Latsi and Kynigos pointed out, we observed that the pupils' construction processes could be divided in two categories in [6]: construction processes through an intrinsic perspective and construction processes through and extrinsic perspective, depended on the point of focus and the way the simulated 3D space was experienced. This division rather reflects the two dominant perspectives people take on space [7]: an external one when they observe space and they manipulate objects in it and an internal one when they explore an environment and when they navigate in it.

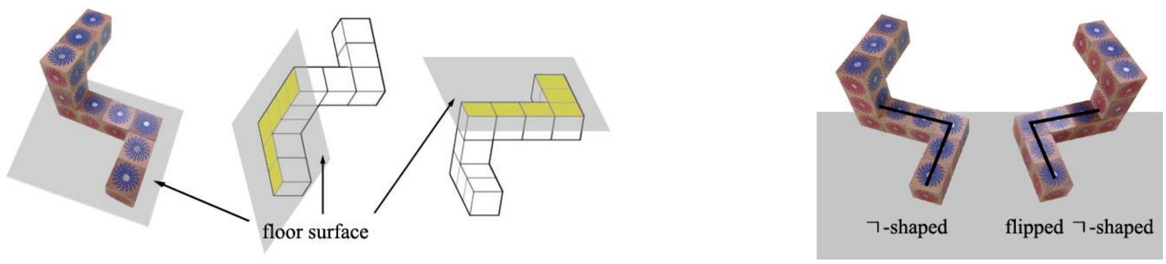
airplane made in virtual world using turtle representation | airplane printed by 3D printer

Figure 4. Airplane made in both virtual and real world

When we just look at the 3D object like in Figure 4, we generally view it from the external perspective. When we are supposed to construct the object using the tool of turtle agent, however, we would change our perspectives from external to internal. That is, we would exam the structure of the airplane carefully, consider which part of the plane we should start with and on which side we should put down the plane, and so on. Like this, in order to construct a 3D object by turtle representation and print it out through a 3D printer, the learner should regard the object not as a holistic product, but as a sequential process he should expore getting on the turtle vehicle. Based on this observation, 3D construction activity was designed and applied to the students with emphasis on the following directions related to the turtle perspective.

First, the students were required to regard the 3D cube stacks as a sequential composition process. Given a 3D object on the monitor screen, the students were asked to write down the executable expressions for the given object in a sequence of their own choice using the 3d turtle representation. Then they were asked to build the object with physical cube blocks with their own hands. Also, students were grouped in teams and in each team, one student explained the shape of a 3D stack and other students attempted to build up physical magnetic cube stacks according to what they understood. When building a 3D object with hands, or explaining the shape to others, or making it listening to the explanation of others, we are to perceive the 3D object as a sequential process, rather than seeing it as a holistic shape. Therefore, this activity was expected to make the determination of 3D object structure more elaborate.

Second, the students were advised to perceive a 3D cube stack considering the floor surface as the reference surface. When printing a 3D object, 3D printers begin with the floor surface and stack layer upon layer to complete the 3D form. In addition, creating 3D stacks in the real world requires a floor surface to place the stack on. It is quite natural to conceive a floor surface to put the object on in the physical world where gravity exists. However, in the virtual space like on the monitor screen, neither gravity nor floor surface exists. This activity showed the students virtual 3D stacks and asked them how they would place the stacks in the real world, which led them to imagine a floor surface on which they would put the object down(see Figure 5a). In order to place the 3D stacks that they built using 3D magnetic stacks, they made the ㄱ-shaped or flipped ㄱ-shaped blocks go on the floor(see Figure 5b). For the case when the ㄱ-shaped block was fixed on the floor and the students were asked to stack the rest of the blocks, students had to think longer about the structure of the stack because the floor surface could not be manipulated freely. Through this activity, students were able to visualize a virtual floor surface and look for the ㄱ–shaped or flipped ㄱ-shaped block when they saw the 3D stacks on the screen. From the conventional method of heuristically approximating similarities and differences of 3D objects, we seek to provide a systematic approach of analysis and categorization that will bring about change in the way we perceive 3D objects. To evaluate the effects of this activity, pre and post MRT were conducted.



a.floor sufaces of 3D object in both real and virtual world   b. ㄱ-shaped and flipped ㄱ-shaped blocks

Figure 5. Floor surfaces of 3D object and type of blocks

4. Experiment

4.1. Participant

Space perception to recognize, represent, transform 3-dimensional object does not require prior knowledge; thus, diverse age groups can be studied. In this experiment, sixty-nine 6th graders (37 male and 32 female students) participated; however, 3 outliers whose pre and post MRT scores are outside the average by 2 standard deviation were excluded, resulting in the total data of 66 students (35 male, 31 female).

4.2. Materials and Stimuli

Using designed MRT items, all participants were pre and post-tested before and after activity. The two tests included the same items. The MRT used in this study is based on analytic research on the existing MRT item in [8][9], and included 12 items based on rotation axis (x, z, xz) and rotation degree (120, 180), (see Table 1). The item stimuli are from the library of Peters& Battista in [10].

item number	rotation axis	rotation degree	item number	rotation axis	rotation degree
1	x	120	7	z	120
2	x	180	8	z	180
3	x	120	9	xz	120(40+80)
4	x	180	10	xz	180(120+60)
5	z	120	11	xz	120(60+60)
6	z	180	12	xz	180(120+60)

Table 1. MRT item characteristics

4.3. Procedure

Sixty-nine students were divided into 3 groups with 23 students in each group. They conducted handwritten pre-test and participated in activity with the researcher for 75 minutes each day for two days followed by post test. The students solved 3 example items before the test with the researcher so that they are familiarized with the test. The students were instructed to mark the box “same” if the two objects looked same when rotated, and mark “different” if they perceived the two objects differently. The test time limit was 4 minutes.

4.4. Hypothesis

Based on Shepard & Metzler’s study in [11] that the response time increases relative to rotation degree when conducting MRT, it was revealed that when rotation is mentally simulated when we rotate mentally as we physically rotate. This enables us to predict that the relative relationship of a pair of object during pre-test, in other words, the accuracy rate can differ based on the rotation direction and degree.

On the other hand, the students are used to seeing 3D stacks using ‘turtle perspective’ which can change the accuracy in the post test depending on how well one understands the ‘turtle perspective’. During the activity, students built 3D stacks by placing ㄱ-shape (turn to the left) or flipped ㄱ-shape (turn to the right) with 3-4 block segment in consideration of gravity; thus, they will base their recognition on the floor surface. The items includes stimuli that change the direction depending on whether the floor surface is looked from the fixed observer’s perspective or from the turtle perspective. For example, let’s think about the second stimulus of Figure 6. The floor surface with ㄱ-shape made up of 3-4-blocks segment is upside down. If the 3D stack is built from the end of the floor surface, then the 4-block segment is left-bound from the fixed observer’s point of view. However, from the turtle perspective, the turtle enters from the back of the floor surface and moves right to build the 3D stack. The remaining stimuli in Figure 6 shows difference in the direction of 4-blocks segment which changes depending on whether it was viewed from the turtle or fixed observer’s perspective. Thus, we created a hypothesis if the stimulus that changes the direction depending on the perspective is included in the item; then it will have lower correct answer percentage than the one without it. The items with such stimulus are 1, 3, 7, 8, 10, and 12.

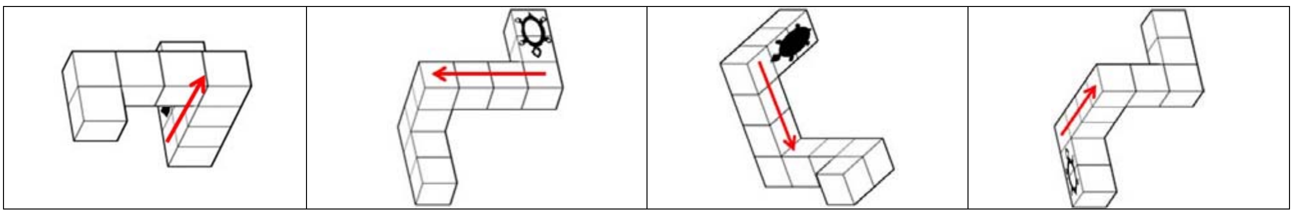
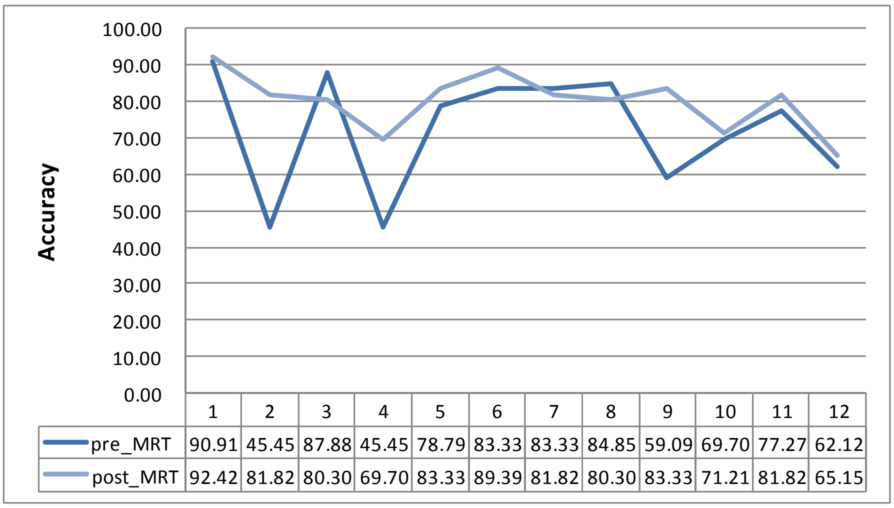


Figure 6. Stimuli that change the direction depending on the turtle perspective

4.5. Result

Based on the pre-test result, the accuracy average of each item depending on the rotation direction was: 67.42% for x-axis rotation, 82.57% for z-axis rotation, and 67.05% for xz-axis rotation. z-axis rotation which only had left-right rotation had more right answers than x-axis or xz-axis which has top-bottom rotation. The accuracy average of each item depending on the rotation degree was: 75.5% for 120°, 65.15% for 180° meaning that 180° had less correct answers than 120°. Such result gives an overview explanation of our hypothesis. Also, the result graph of each item in Figure 7 shows that the difference created by rotation degree changes with the rotation direction. For example, in items 5, 6, 7 and 8 for z-axis rotation, overall high accuracy rate was shown with minor differences. However, in x and xz-axis rotation item, the accuracy differed vastly depending on the item.

In the post test, we expected items 1, 3, 7, 8, 10, and 12 to have low accuracy since they include stimulus that changes the direction depending on perspective. The post-test accuracy in Figure 7, items 1 and 6 shows high rate of above 85% whereas items 4, 10, 12 has low rate of below 70%. Items 10 and 12 yielded results that align with our prediction; however, item 1 shows the opposite. Our hypothesis was not confirmed; thus, we had to analyze the relative changes in the accuracy of pre and post-tests in consideration of characteristics of items rather than the absolute percentage of accurate answers of pre and post-tests in order to assess the effect of the activity. The change trend of pre and post-tests is categorized into 4 groups as follows.

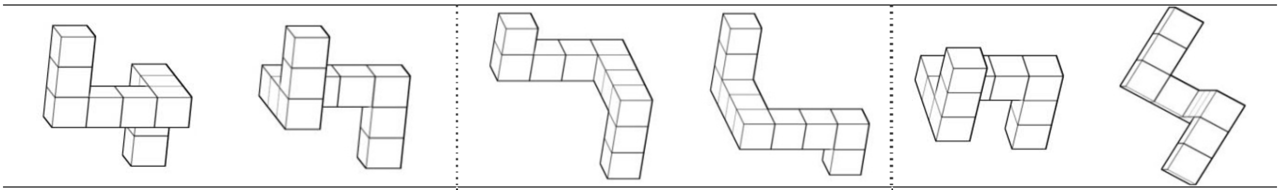


	1	2	3	4	5	6	7	8	9	10	11	12
pre_MRT	90.91	45.45	87.88	45.45	78.79	83.33	83.33	84.85	59.09	69.70	77.27	62.12
post_MRT	92.42	81.82	80.30	69.70	83.33	89.39	81.82	80.30	83.33	71.21	81.82	65.15

Figure 7. Pre and post MRT result and its graph

4.5.1. Items with Significant Increase in Accuracy

The pre and post-tests accuracy graph in Figure 7, a significant hike is observed in items 2, 4, and 9 in terms of accuracy. We tested the difference for each item using paired t-test to assess pre and post test accuracy, and the percentage significantly increased only in item 2 ($t=-5.42$, $p<.001$), 4 ($t=-3.38$, $p<.001$), and 9 ($t=-3.23$, $p<.05$) in post-test from pre-test. The analysis on the characteristics of items 2, 4, and 9 of Figure 8 showed that all three items involved top-bottom rotation which explains low accuracy in mental rotation. On the other hand, the direction is the same for observer and turtle for stimuli; thus, it explains why the accuracy increased when simulation is used for turtle perspective. In other words, these items serve an alternative to heuristic approach for mental rotation, and it can be assessed that this group can reduce the problem-solving errors by adopting 'turtle perspective'

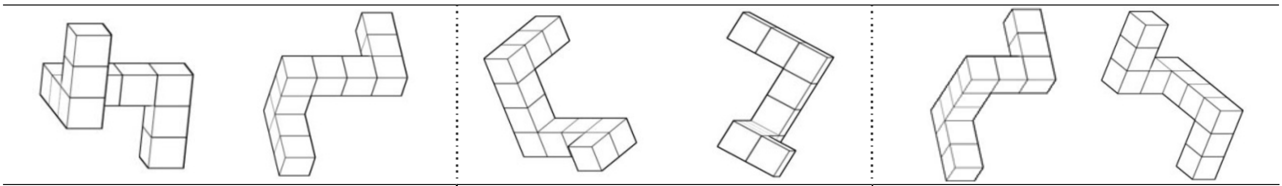


item 2	item 4	item 9

Figure 8. Stimuli of items increased in accuracy significantly

4.5.2. Items with Significant Decrease in Accuracy

The Figure 7 graph shows that post-test accuracy increased in most items except for in items 3, 7, and 8. We analyzed the characteristics of these three items in Figure 9 of which result revealed that item 3 is 120° rotation and items 7 and 8 only include left-right rotation which could lead to estimation that it was not difficult for students to mentally rotate. However, all three items include stimulus which changes the direction depending on perspective; thus, this type can be more prone to errors when they are characterized from the turtle perspective than when the two objects are heuristically rotated.



item 3	item 7	item 8

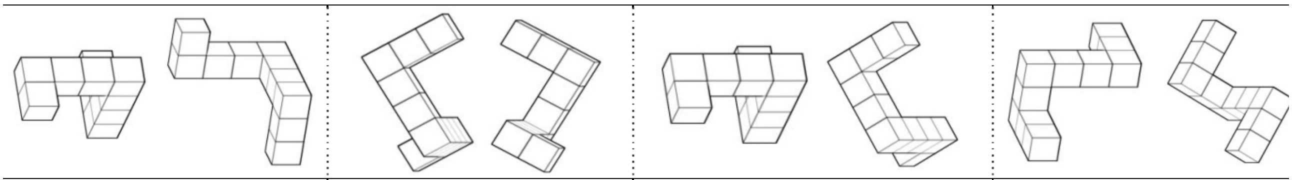
Figure 9. Stimuli of items decreased in accuracy

4.5.3. High Accuracy Items in Both Tests

Some items show high accuracy in both pre and post test. Figure 7 graph shows that items 1 and 6 shows high accuracy 80~95% in both tests. We analyzed the characteristics of items 1 and 6 in Figure 10 and it revealed that high accuracy was shown in pre-test in which students mentally rotate. Item 1 involved 120° rotation and item 6 involved left-right rotation. However, although item 1 includes stimulus that changes its direction from the turtle perspective, it still yielded high accuracy in post-test. If our hypothesis proved valid, then the accuracy of item 1 should have dropped in post-test; however, the result showed high accuracy in both pre and post-tests. Item 6 does not include such stimulus which can cause high accuracy in post-test.

4.5.4. Low Accuracy Items in Both Tests

Some items show low accuracy in both pre and post-tests. Figure 7 graph shows that items 10 and 12 resulted in very low accuracy of 60~70% in both pre and post-tests. The analysis of items 10 and 12 in Figure 10, they both include top-bottom as well as 180° rotation of xz-axis which could increase errors in mental rotation. Additionally, they both include stimuli that change direction depending on perspective; thus, they are again prone to more errors from the turtle perspective. These items can be categorized as error-prone group both when heuristically rotated as well as when each item is characterized from the turtle perspective.



item 1	item 6	item 10	item 12

Figure 10 . Stimuli of items high or low accuracy items on both test consistently

Through the pre and post MRT results, the item of whether the 3D object recognition and structure differentiation heuristic approach can be mediated by the activity designed based on the turtle metaphor was investigated. The mediation that occurs between the pre and post assessments is not simply drill and practice but holds significance in that it is an educational activity designed upon two principles.

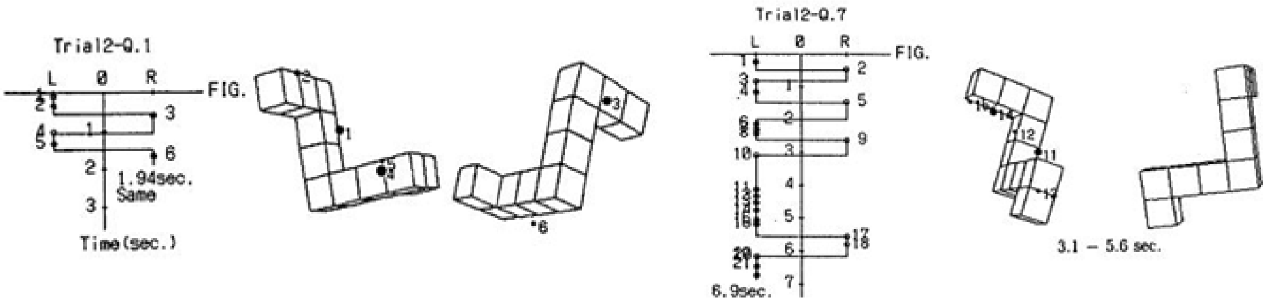
First, the designed activity involves sequential process construction rather than recognition of a 3D object as a holistic product. Similarly to when a young child becomes accustomed to the sequential ordering number intuitively when learning about the number concept then absorbing the concept of cardinal numbers as sets, the order or sequence can act as an entry point for novices taking in a new concept. In this regard, the sequential approach to 3D objects holds educational significance as the entry point to recognizing and representing 3D objects.

Second, through the activity, students were able to reinterpret 3D stacks to a more comfortable subject by making a virtual floor surface as the reference. Thus, structurally similar isomorphism was found by approaching two seemingly different mental images through the mental models of the ㄱ-shaped block and flipped ㄱ-shaped block. Landriscina found that mental models interacted with mental simulation and relatively stable cognitive structure at times overlapped with that of 'schema' [12]. In other words, the educational value of the activity was obvious as the students naturally differentiated between the ㄱ-shape type and flipped ㄱ-shape type within the activity which acted as the mental model in viewing the 3D object.

Lastly, the designed activity can be described as an embodied activity in the sense that the learner directly composes the 3D object through their sensory motors in a physical world and mentally simulates the transformation of the 3D object with the body-syntonicity. From the perspective of embodied cognition where the human cognition is a result of interactions between the brain, body, and environment, this activity holds educational value as it is an embodied activity that aids the human cognitive process regarding the representation and transformation of 3D objects.

5. Closing Remarks

Humans make judgments heuristically based on sense and intuition when decision making is necessary under uncertainties. The dependency on instinct is most clear especially when there is no cognitive tools to represent and manipulate a subject. Therefore, different people may perceive the same object differently according to their personal experiences, goals, and strategies. Based on the fact that there is no appropriate cognitive tool to represent and manipulate 3D objects, this study presented the 3D turtle representation system for the expression and communication of 3D objects. Activities were designed and conducted to examine whether such activity could mediate the heuristic cognitive process of humans for 3D cube stacks and the result was confirmed through MRT. Each questions could be categorized into one of four types based on the accuracy rate change discovered in pre and post test. Further analysis has revealed that in the pre evaluation, students solved problems by rotating the objects mentally depending on their sense while in the post evaluation, they solved the problems using the turtle perspective through embodied simulation. The results of the experiment showed that the designed 3D construction activity altered the 3D object cognitive process. In this study, the pre and post MRT answer accuracies were compared according to the item characteristics to infer the cognitive mechanism used by the students and confirm the hypothesis. However, because the cognitive mechanism involved in solving the problems is very complex, there are aspects that cannot be explained through the hypothesis of this study. Therefore, a more thorough and scientific approach will be necessary in future studies. For example, Kumiko Shiina et al. used an eyetracker during the MRT as shown in Figure 11 to observe the eye movement of the subject students and categorize the cases by patterns [13]. These patterns will be different depending on the strategy and perspective explained in this study. Using an eyetracker or EEG based on the results of this study to analyze the cognitive mechanism occurring during the tests, the elements which need further investigation in this study may be identified and explained with improved understanding.



r-pattern f-pattern

Figure 11. Pattern of eye movement using eyetracker (Kumiko Shiina et al.,1997)

6. Acknowledgements

This paper is supported in part by the educational projects between Seoul National University and Si-Heung City in Korea

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