ARTIFACTS: INFLUENCING PRACTICE AND SUPPORTING CREATIVITY PROCESS IN THE MATHEMATICS CLASSROOMS

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Abstract

The study presented here is part of an ongoing research project on the relationship between everyday mathematics, in particular the numerical culture children acquired outside the school, and classroom mathematics, and the ways each can inform the other in the development of abstract mathematical knowledge. In this study we wanted in particular to investigate: i) the primary school students' capacity to create and deal with mathematical problems; ii) the potential that the problem posing activity has for identifying and stimulating creative thinking in mathematics, when problem-posing process is implemented in meaningful situations involving the use of suitable real-life artifacts; iii) the potential that the problem solving activity has for fostering a reflection on a previous problem posing activity and on the nature of problems created by students themselves, and iv) the primary school students' capacity to solve mathematical problems, to discuss their structure and to describe the reasoning behind the solutions they offered.

Keywords problem posing, problem solving, creativity, artifacts, primary school

1. Introduction

We maintain that the problem-posing process represents one of the forms of authentic mathematical inquiry which, if suitably implemented in classroom activities, could move well beyond the limitations of word problems, at least as they are typically utilised. Furthermore, problem posing could help students to prepare to cope with natural situations they will have to face out of school.

> Almost all of the mathematical problems a student encounters have been proposed and formulated by another person – the teacher or the textbook author. In real life outside of school, however, many problems, if not most, must be created or discover by the solver, who gives the problem an initial formulation ([1], p. 124).

Furthermore, there is a certain degree of agreement in recommending problem-posing and problem-solving activities to promote creative thinking in the students and assess it. [2] argued that creativity lies in the interplay between problem posing and problem solving processes: it is in this interplay of formulating, attempting to solve, reformulating, and eventually solving a problem that one sees creative activity. Both the process and the products of this activity can be evaluated in order to determine the extent to which creativity is evident. Among the features of this creative activity that one might examine there are the novelty of the problem formulation and the problem solution.

Creativity, understood as the cognitive ability to create and invent, is linked to the activity of mathematical problem posing. In fact, problem posing is a form of mathematical creation: the

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creation of mathematical problems from a specific context, as meaningful products ([3]). Furthermore, problem posing is a form of creative activity that can operate within tasks involving structured *rich contexts* ([4]), using real-life artifacts and human interactions.

The study presented here is part of an ongoing research project on the relationship between everyday mathematics, in particular the numerical culture children acquired outside the school, and classroom mathematics, and the ways each can inform the other in the development of abstract mathematical knowledge. The focus is on fostering a mindful approach toward realistic mathematical modelling, as well as a problem-posing attitude ([5]). The project aimed at showing how an extensive use of suitable artifacts, with their incorporated mathematics, a variety of complementary, integrated, and interactive teaching methods, and the introduction of *new socio-mathematical norms* ([6]), can play a role in order to create a substantially modified teaching/learning environment. These socio-mathematical norms are constructed and continually modified through the interaction between teacher and pupils, as well as by the artifacts, whose introduction into the classroom setting brings from the outside world potential norms and ways of reflection that open lines of cultural conceptual development to the children ([5]). The development of mathematical reasoning and sense-making processes is seen as "inseparably interwoven with their participation in the interactive constitution of taken-as-shared mathematical meanings and norms" ([6]).

In the study here presented we wanted in particular to investigate: i) the primary school students' capacity to create and deal with mathematical problems; ii) the potential that the problem posing activity has for identifying and stimulating creative thinking in mathematics, when problem-posing process is implemented in meaningful situations involving the use of suitable real-life artifacts; iii) the potential that the problem solving activity has for fostering a reflection on a previous problem posing activity and on the nature of problems created by students themselves, and iv) the primary school students' capacity to solve mathematical problems, to discuss their structure and to describe the reasoning behind the solutions they offered.

2. Theoretical and empirical background

2.1 Mathematical modelling

The term mathematical modelling is not only used to refer to a process whereby a situation has to be problematized and understood, translated into mathematics, worked out mathematically, translated back into the original (real-world) situation, evaluated and communicated. Besides this type of modelling, which requires that the student has already at his disposal at least some mathematical models and tools to mathematize, there is another kind of modelling, wherein model-eliciting activities are used as a vehicle for *the development* (rather than the application) of mathematical concepts. The '*emergent modelling'* approach ([7]) taps into the second type of modelling, and its focus is on long-term learning processes, in which a model develops from an informal, situated model ("a model of"), into a generalizable mathematical structure ("a model for").

> These emergent models are seen as originating from activity in, and reasoning about situations. From this perspective, the process of constructing models is one of progressive reorganising situations. ([7]).

Although it is very difficult, if not impossible, to make a sharp distinction between the two aspects of mathematical modelling, it is clear that they are associated with different phases in the teaching/learning process and with different kinds of instructional activities ([8]). However, at primary school level, the focus will be more addressed to the second aspect of mathematical modelling, in particular the construction of a model can be considered as a meaningful product ([9]). Further we will argue for modelling can be seen as a means of recognizing the potential of mathematics as a critical tool to interpret and understand reality, the communities children live in, or society in general ([10]).

2.2. Problem posing

It is well recognized that problem posing is an important component of the mathematical curriculum and, indeed, lies at the heart of mathematical activity ([11]). Not surprisingly, reports such as those produced by the National Council of Teachers of Mathematics have called for an increased emphasis on problem-posing activities in the mathematics classroom.
Problem posing and problem solving are closely related. As [12] suggested, problem posing could occur prior to problem solving when problems were being generated from a particular situation or after solving a problem when experiences from the problem-solving context are modified or applied to new situations. In addition, problem posing could occur during problem solving when the individual intentionally changes goals while in the process of solving the problem.
Despite its significance in the curriculum, problem posing has not received the attention it warrants from the mathematics education community. Little is known about the nature of the underlying thinking processes that constitute problem posing, and the schemes through which students' mathematical problem posing can be analyzed and assessed ([13]). We know comparatively little about children's ability to create their own problems in both numerical and non-numerical contexts or about the extent to which these abilities are linked to competence in problem solving. We also have insufficient information on how children respond to programs designed to develop their problem-posing skills ([12]). Research on these issues is particularly warranted, given the well-documented evidence that young children's creativity and open-mindedness in generating and solving problems dissipate as they progress toward the higher school grades ([11]).
Problem posing has been defined by researchers from different perspectives (see [14]). In this contribution we consider mathematical problem posing as the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems. In this case the problem created by the children is a meaningful in the constructionism sense ([9]).
This process becomes an opportunity for interpretation and analysis of reality in different ways: i) they have to distinguish significant data from irrelevant data; ii) they must discover the relations between the facts; iii) they must decide whether the information in their possession is sufficient to solve the problem; and iv) they must investigate if numerical data involved is numerically and/or contextually coherent. These activities, quite absent from today's school context, are typical of the modelling process and can help students to prepare to cope with natural situations they will have to face out of school.

2.3. Creativity

[15] argued that the classical school mathematical problems like "Kate has seven apples. She gives four apples to John, how many apples has Kate now?", presented by the teacher, solved by a specific method, and evaluated by the teacher from a prepared answer key, have little to do with creativity. Mathematics is about raising questions as well as answering them, finding new relationships and generalizing old ones. The ability to shape (problem posing process) and solve (problem solving process) mathematical problems, allowed students to construct mathematical meaning, is the core of the mathematical thinking.

So, problem-posing and problem-solving activities are used to promote creative thinking in the students and assess it. Silver and other authors (see [16], [12], [17], [18] and [19]) have linked problem posing skills with creativity, citing flexibility, fluency, and originality as creativity categories.
Creativity has received much attention in the literature, particularly in relation to distinctions between two types of thought: productive (divergent thinking) and reproductive (convergent) thinking ([20]). One of the main lines of research on creativity concerns exactly the distinction between these two types of thought. First, Guilford dealt with creativity and noted that IQ and creativity could not be overlapped. He therefore hypothesized that a person could be creative without exceptional intelligence and vice versa. Thus, creativity began to be recognized as an asset, even if present in different degrees and shapes, of each person. Guilford saw creative thinking as clearly involving what he categorized as divergent production. He broke down it into nine skills: sensitivity to problems, ideational fluency, flexibility of set, originality, the ability to synthesize, analytical skills, the ability to reorganize, span of ideational structure, and evaluating ability. All these skills influence each other and represent the related aspects of a dynamic and unified cognitive system. The main models that describe the mathematical creative process emphasize the importance of sensitivity to the problems (problem finding) and their resolution (problem solving). Problem finding, in particular, may be associated with mathematical problem posing; and the problems created are the products of a creative process.
[2] argued that problem-solving and problem-posing tasks and activities could assist students to develop more creative approaches to mathematics: they can increase students' capacity with respect to the core dimensions of creativity, namely, fluency, flexibility, and originality. In [19], a recent study, the authors used these parameters as indicators of creativity in students' problem posing. We believe in the didactic potential of using suitable artifacts, combined with particular teaching methods, as a source for these types of tasks and activities.

2.4. Artifacts

Tools, artifacts, and cultural representational systems are important components of mathematical learning. Several educators (e.g. [21], [22] and [23]) have noted that is not the artifact (or tool) in isolation that offers support to the teacher – rather the student use of the tool and the meanings they have developed as a result of the activity ('*cognitive activity is not limited to the use of tools or signs',* [21]). The mathematical goals emerge for children not only in relation to artifacts but even in relation to structure of activity, social interaction and children's prior understanding ([23]).

> Given the complex interaction between the use of the tools and the development of reasoning and learning, the question that should concern educators is not how powerful or effective cultural tools are in promoting learning, but rather what teaching practices and classroom interactions can promote meaningful learning and understanding of the mathematical principles and relations embedded in cultural tools and representations ([22], p. 302).

The use of real-life artifacts in our classroom activities has been articulated in various stages, with different educational and content objectives (for a description see e.g. [10]). In some cases we introduced into classroom activities materials, real or reproduced, which children typically meet in real-life situations, for example supermarket bills, bottle and can labels, a weekly TV guide, the weather forecast from a newspaper, some menus of restaurants and pizzerias (see e.g. [10] and [24]). In other cases the artifacts have been constructed by children, for example a calendar ([25]) or the plan of a detailed journey with the help of Internet. In [26] the proposed activity consisted in asking the children of fifth year of primary schools to plan a detailed journey with the help of Internet; in particular one of the tasks (done under the form of a story/cartoon strip in English with protagonist the class teacher) consisted in finding the cheapest travel mean for reaching Bratislava (by consulting websites suggested by the teacher). The activity isn't so easy since the school is on Lago di Garda, a location very close to Verona airport as well as Venice airport, and also the three airports in Milan are easy to reach. These conditions created a greater difficulty in the calculation of the cheaper travel since the students had to calculate a series of possible combinations.

Besides the use of suitable artifacts the teaching/learning environment designed and implemented in our mathematics classroom activities is characterized by the application of a variety of complementary, integrated and interactive instructional techniques (involving children's own written descriptions of the methods they use, work in pairs or small groups and whole-class discussions) in an attempt to establish a new classroom culture also through new socio-mathematical norms ([5]).

The idea is not only to motivate students with everyday-life contexts but also to look for contexts that are experientially real for the students and can be used as starting points also for progressive mathematization, in order to favour also a mathematical modelling disposition.

3. The study

The overall aim of this exploratory study was to examine the relationship between *problem-posing* and *problem-solving* activities and *creativity*, when problem-posing process is implemented in situations involving the use of real-life artifacts, with their embedded mathematics. In this study, Guilford's characterization of creativity has been adopted.

In particular the study sought to investigate:

- the role of suitable artifacts as sources of stimulation for the problem posing process,
- the primary school students' capacity to create and deal with mathematical problems (including open-ended problems),
- the potential that the problem posing activity has for identifying and stimulating creative thinking in mathematics,
- the potential that the problem solving activity has for fostering a reflection on a previous problem posing activity and on the nature of problems created by students themselves,
- the primary school students' capacity to solve mathematical problems, to discuss their structure and to describe the reasoning behind the solutions they offered,
- a method for analyzing the products of problem posing that the teacher could use in the classroom to identify and assess both the activity of problem posing itself and the creativity of the students.

This exploratory study involved four fifth-grade classes (10-11 years old) from two primary schools in northern Italy, for a total of 71 pupils. The study was carried out by the second author in the presence of the official logic-mathematics teacher. The first primary school was located in an urban area situated within a few miles of the centre of a city. The children were already familiar with activities using real-life artifacts, group work and discussions. The second primary school was located in a mountainous area. The children were not already familiar with these types of activities, even though the teacher had once proposed a problem-posing activity where the situation was a drawing of the prices of different products in a shop.

The artifact used was the page of a brochure containing: the special rates for groups visiting the Italian amusement park "Mirabilandia", the menu and discounts applied, the cost for access to the beach, etc. This artifact was chosen considering that all students were already familiar with an amusement park because they had been to one. This page was full of information, including prices (some expressed by decimal numbers), percentages, and constraints on eligibility for the various offers. We wanted to motivate students by offering them a semi-structured situation as rich and contextualized as possible in order to permit them to use their extra-scholastic experience in the creation and resolution of problems.

3.1 Teaching experiment

The experiment consisted of three phases: (1) the presentation of the artifact used; (2) a problem-posing activity; (3) a problem-solving activity. The activities took place in three different days, a few days apart. The students worked individually for part (2); for part (3) they at first were divided in couples or in groups of three students and then participated in a collective discussion. Students could use the artifact and its summary during all three activities.
The first phase, of about two hours, consisted in the analysis and synthesis of the artifact. This phase was preparatory to the problem-posing activity. After presenting the whole brochure, a copy of one of the pages was given to each student and then he/she was invited to write down everything they could see on that page. Following that, there was a discussion on the observations: the aim was to verify students' understanding of the artifact and to create a summary of the mathematical concepts involved.
The second phase, lasting about an hour, consisted of an individual problem-posing activity in which the children had to create the greatest number of solvable mathematics problems (in a maximum time of 45-50 minutes), preferably of various degrees of difficulty, to bring to their partners in the other classroom. The children were not informed of the time limit in order to avoid them from experiencing anxiety. Rather, they were told that they would have plenty of time to do this activity and that problems would be collected when the majority of the students had finished. To allow for the pupils' self-assessment, they were given a sheet of paper for their calculations and solutions to the problems they had invented. Then, four problems for the next problem-solving activity were selected from among all the problems that had been created. In every class the problems chosen were problems with insufficient data, problems with an incorrect data or articulated problems in order to favor a discussion amongst the students. The following are three problems created by the children and chosen for the next phase.
A group of 20 people, children and adults, decide to go to Mirabilandia. In total, they spend 480 euro. How much will each person pay to enter? (480 euro is an incorrect data because all of the conditions of the artifact were not taken into account, in fact for every 10 entries, 1 entry was free).
Luca and his 10 friends go to Mirabilandia to celebrate Luca's birthday. How much did they spend? (problem with insufficient information),
John decides to celebrate his birthday in Mirabilandia. Overall there are 10 people, 6 adults and 4 children. The 3 children pay 26 euro each one and the adults pay 31 euro each one; John, because it is him birthday, he doesn't pay. The people decide to make a refreshing that costs 10 ,50 euro (per person). How much do they pay for everything? (multistep problem).
The third phase, lasting about two hours, consisted of a problem-solving activity by students and ended with a collective discussion. The students were asked to solve problems, to write the procedure that they had used and to write considerations on the problem itself. Different problem solutions and ideas about problems structure that emerged during the discussion were compared and, at the end of the activity, a collective text summarizing the students' conclusions was written. So at this phase, a reflection on the previous problem posing activity and on the nature of problems invented was favoured by problem solving activity.

3.2 Methodology and data analysis

Data from the teaching experiment included the students' written work, fields' notes of classroom observations and audio recordings of the collective discussions. All of the problems created by the students were analyzed with respect to their quantity and quality. To analyze the type of problems

invented we followed the methodology proposed by [27]; as regards to the analysis of the text of the problems we referred to the research of [14] and [18].
The plausible mathematical problems (in the sense that they can apparently be solved, with no discrepant information, and with respect the conditions in the artifact) with sufficient data were analyzed with respect to their complexity, and were assessed from two perspectives: the first assessed the complexity of the solution and the second assessed the complexity of the text of the problem. With regard to the complexity of the solution, these mathematical problems were divided into multi-step, one-step and zero-step problems. With regard to the complexity of the text of the problems, plausible mathematical problems with sufficient data were divided into problems with a question and problems with more than one question. The latter, were divided in concatenated questions and non-concatenated questions. Furthermore, only the plausible mathematical problems with sufficient data were re-analyzed, this time, to evaluate their creativity. The criteria used to classify them were: the number and type of details extrapolated from the artifact, the type of questions posed, and the added data included by the students.
To evaluate their creativity in mathematics, three categories were taken into consideration—fluency, flexibility, and originality—as proposed by Guilford to define creativity, and as used in the tests by Torrance and in other studies such as that by [19].
When considering the fluency of a problem, the total number of problems invented by the pupils of each school in a given time period, as well as the average number of problems created by each student, were taken into account.
Flexibility, instead, refers to the number of different and pertinent ideas created in a given time period. In order to evaluate the flexibility of the students, the mathematical problems were categorized considering both the number of details present in the brochure (e.g., entrance fee, price of lunch, etc.) which were incorporated into the text of the problem posed, and the additional data introduced by the students (e.g., calculating the change due after a payment). Once the problems had been categorized in the above way, the various types of problems that occurred in each class were counted.
The originality of the mathematical problems created by the students took into consideration the rareness of the problem compared to the others posed in each school. In order to evaluate the originality of a problem, it was considered original if it was posed by less than 10% of the pupils in each school ([18]).

3.3. Some results and comments

A total of 63 students in both schools participated in the problem-posing phase and they created a total of 189 problems. Students from the first school invented 58 problems (57 were mathematical problems), while students from the second school invented 131 (all mathematical problems).
More than half of the problems that were invented by the students are solvable mathematical problems (64% of the problems created by the pupils of the first school and 60% of those created by the pupils of the second school). Table 1 shows the main quantitative results of both schools:

Category	First School	Second School
Non mathematical problem	**1,72 %**	**0%**
Implausible mathematical problem	**18,97 %**	**29%**
Plausible mathematical problem with insufficient data	**8,62 %**	**10,69%**
Plausible mathematical problem with sufficient data	**63,79 %**	**60, 31%**

Table 1: The main quantitative results of both schools

Most of the problems created were similar to the standard ones used in schools, although there were some cases of creative and open-ended problems. The following is an example of creative problem: *A group of 30 school children go to Mirabilandia. Every child should pay 20 euro, but today is the birthday of Paola, who then receives a discount of 50%. At the lunch all the group take a Pizza Time that cost 7.50 euro. How much does the group spend?*

Then, analyzing these solvable mathematical problems we have found that i) the 81% of the first school and the 75% of the second school are multi-step problems; ii) the 78% of the first school and the 73% of the second school are problems with a question. Regarding problems with more than one question, in the first school the 62% have concatenated questions, and about 43% in the second school.

As far as creativity is concerned, the second school was more successful in all three categories used to assess performance (fluency, flexibility and originality). With regard to fluency, each student in the first school invented 2 problems on average, while each pupil of the second school invented 3 problems on average. With regard to flexibility, the problems created by the classes of the first school were divided into 11 categories, those of the second school into 16 categories. In evaluating originality, it was found that 3 original problems were created in the first school and 10 in the second school. Original problems include inverse problems, and problems containing almost all the information of the artifact.

It was thus found that the students of the second school demonstrated better performance on the suggested problem-posing task, in terms of creativity indicators, than the students of the first school, even though the students from the first school were familiar with posing problems from artifacts from their past study.

With regard to the problem-solving phase, this appears to be important and helpful in allowing a better understanding of the initial situation, fostering quality control of the problems created by the students themselves, and giving a starting point for analyzing the structure of problems.

By solving the problems created by their peers, the students become able to analyze them in a more detached and critical way. For example, students reflected on what information was really important and what was not, and discovered that numerical information is not always the most important information contained in the text of a problem, as the following discussion illustrates:

> It's Giulia's birthday and she invited 9 people to her birthday party, but she didn't benefit from the "pacchetto festa" (party package), how much did Giulia pay for her entrance?

Almost all of the students did not read the words of the problem question carefully, because a lot of students calculated the total and not only Giulia's entrance cost. In fact the total number of people (9) in the problem was superfluous.

The problems solved by the children of the first school allowed them to make estimate: the children have thought about what happens in everyday life in order to estimate the possible solution of the problem. Presenting children problems with insufficient data has allowed them to better understand the artifact and the text of the problem itself.

The results obtained, checked also in another study we conducted (see e.g. [10] and [6]), show that an extensive use of suitable cultural artifacts, with their associated mathematics, can play a fundamental role in bringing students' out-of-school reasoning and experiences into play by creating a new dialectic between school mathematics and the real world.

With regard to the artifact utilized, on the one hand it was particularly attractive inasmuch as it referred to an amusement park known to the children, while on the other hand it was also very dense with information. The latter aspect was desirable because it furnished conditions allowing students to formulate hypotheses regarding the various possibilities offered. Students were therefore able to create diverse problems with differing degrees of difficulty. However, the artifact also revealed some weak points: it was rather complicated in its language and presented some information only implicitly. Students, in fact, did not know some terms of everyday language (for

example lunch voucher, one free entry every 10 entries) and several mathematical terms (for example minimum, at least).
Despite the richness of variables and linguistic limits of the artifact, the majority of the pupils nevertheless succeeded in understanding it and in carrying out both the problem-posing and problem-solving activities. Moreover, the brochure's complexity improved lexical enrichment, favoured the development of interesting discussions (which we did not report here), and stimulated the students to ask themselves questions and formulate hypotheses, that is to "problematize the reality".
However, in order to have better performance on the problem-posing task in terms of the greater number of plausible problems, with more complex tests and concatenated questions, it proved to be important to structure, organize, and summarize the information present in the brochure. In fact, students who had previously performed this type of analysis outperformed the others in the problem-posing activity. With regard to this aspect, the first school students, who were already familiar with this type of activity, have done better analysis and synthesis of the artefact by producing fewer implausible problems. Instead, about one-third of the problems produced by the second school students were implausible problems.

4. Conclusion and open problems

Problem posing is of central importance in the discipline of mathematics and in the nature of mathematical thinking, and it is an important companion to problem solving ([1]). The advancement of mathematics requires creative imagination, which is the result of raising new questions, new possibilities, and viewing old questions from a new angle.

Researchers have slowly realized that developing the ability to pose mathematics problems is at least as important, educationally, as developing the ability to solve them.

> Problem formulating should be viewed not only as a *goal* of instruction but also as a *means* of instruction. The experience of discovering and creating one's own mathematics problems ought to be part of every student's education. Instead, it is an experience few students have today – perhaps only if they are candidates for advanced degrees in mathematics ([1], p.123).

The study here presented shows the importance of combining the problem posing activity with the problem solving activity: the problem-solving phase, in fact, combined with group discussions, allowed students to reflect on different types of problems and explore new possibilities (e.g. suggesting that mathematical problems do not always require a numerical answer or a unique solution, and that there are problems which are not solvable). Furthermore, the results of the classroom discussion suggest that asking students to analyze the problems they have created facilitated their critical thinking because students felt free to discuss the validity of the problem, to consider different assumptions, and to decide whether a problem had been solved or not. Finally it is interesting to reflect on the fact that in the study here presented there were good results for students accustomed to using real life artifacts (the classes from the first school) as well as those who have used them for the first time (the classes from the second school). This indicates that an artifact provides a useful context for the creation of problems and the mathematization of reality as a result of its accessibility to all students.

In future research, we would like to investigate through further studies how classroom teaching practices and experiences influence the creativity processes.

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