

DIGITAL LEARNING ENVIRONMENT FOR INQUIRY-BASED LEARNING OF MATHEMATICS

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Abstract

Digital technologies provide a variety of resources to implement new forms of dialogue and interaction of students with a learning environment. Interactivity, feedback and aimed help allow to implement didactical strategies and tools guiding the learning process into learning materials. This paper describes two interactive learning activities for exploring and discovering mathematical relationships. The first activity focuses on the discovery of divisibility rule for number four. Spreadsheets were used to create a sequence of tasks whose solutions were automatically evaluated. The second activity leads students to examine and reason about the properties of a cyclic quadrilateral. To implement a stimulating learning environment we linked the authoring system ToolBook with the dynamic geometry system Cabri. A graded system of auxiliary information is incorporated into this activity to enhance effective learning. If necessary or if the student gives repeatedly wrong answers, he gets more and more detailed instructions and helpful information to resolve the problem.

Keywords interactive activities, learning of mathematics, feedback, contingent tutoring, divisibility by four, cyclic quadrilateral

1. Implementation of constructionist approaches to interactive learning activities

An important part of designing interactive learning materials for applying constructionist approaches to learning is the implementation of a learning strategy based on a rich educational context and a sequence of appropriate tasks. The interactive activities based on constructionist approaches should contain learning situations that will foster students' independent inquiry and discovery of new mathematical concepts and relationships. Learning situations established in this way should motivate students to learn and should provide them the opportunity to use their previous knowledge to extend their knowledge and improve their ideas about abstract mathematical concepts and relationships.

Students can approach new knowledge in a number of ways. That is why multiple representations and different ways to construct their own understanding of knowledge should be incorporated into learning environments. The use of multiple representations can offer students more opportunities to connect explored objects and relationships to their previous conceptions. Students should discuss together procedures used in discovering new knowledge and try to find appropriate arguments to reason about their findings. By working on meaningful activities, the students explore mathematical context, acquire new skills and are actively engaged in making findings based on which they create new ideas and construct their knowledge.

Feedback and a system of auxiliary information are appropriate ways to guide the learning process. If a student is unable to deal with a problem, the system is able to offer additional questions and advice. When providing help, one can distinguish between executive and instrumental help-seeking [3]. Executive help-seeking leads the student directly to the solution of the problem. Instrumental help-seeking orients student's attention to the information which then can be used to solve the problem and thus leads student to the discovery of new findings. This type of assistance is an important tool for guiding the learning. Depending on the ability of the student, it is appropriate to create a graded system of auxiliary information to aid the solving of the tasks.

We try to design interactive learning activities in which students can experiment with different types of mathematical objects and to get immediate feedback about correctness of their conjectures and solutions. A dynamic geometric system such as Cabri can be used to experiment with geometric objects. This exploratory environment allows students to manipulate objects and investigate invariant properties of geometric shapes and their relationships. We use the authoring system Toolbook together with dynamic constructions to develop interactive activities for using various types of mathematical models and for investigating properties of geometric shapes. For example, a geometric model was used to create interactive activities for optimization problem solving. Interactive activities for investigating triangles, quadrilaterals, and circles were prepared to stimulate an active learning of geometry. As a demonstration, an interactive activity named Cyclic quadrilateral is described in the third section of this paper. In this activity, a student can evaluate the behaviour of objects and compares them with his expectations. Differences between his results and expectations then lead to further investigation.

2. Interactive workbook to discover divisibility rule for number 4

A spreadsheet environment was used to develop several interactive activities for learning functions, statistics, and divisibility. For the application of constructionist approaches to discover rules for divisibility by 4, 9 and 11, we developed interactive workbooks. As an example we chose a workbook for the divisibility by 4. The basis of the workbook is a sequence of eight tasks assigned on separate sheets when solved could lead students to discover the rule for divisibility by 4. By solving these tasks students could gradually find arguments to justify their findings. The findings would then be formulated on the last sheet of the workbook.

Control buttons are placed on each sheet. They provide the evaluation of students' solutions and also allow the student to move to the next task. In most cases this move is conditioned by solving the previous task. Before using the workbook students should have acquired basic knowledge from the topic of divisibility and about the divisibility rule by numbers 2, 5 and 10. In addition to the skills of using these rules in solving tasks, students should also understand why it is sufficient to examine the last digit of numbers. The teacher should assign several tasks to use an unknown character "c", for which any digit can be substituted into the notation of the number. As an example we mention the task: Create a two-digit number with one character "c" so that you get a number divisible by 5 after substituting any digit for "c".

After opening the workbook all sheets except the first one with a motivation problem are automatically hidden. When solving the first problem students should look for remainders after dividing two-digit natural numbers by number 4. For simple calculations students can also use paper and pencil. This sheet is locked and students can write only into the highlighted cells. In this way they will not disrupt the functionality of the dynamic buttons and the formulas used to provide feedback. The main part of the first sheet is shown in Figure 1.

A father bought some school items for his two sons. The table below lists the items and their prices. He decides to distribute the items between his sons. The total price is shared such that the eldest son receives three times that of the youngest son. Before the distribution the father decides to keep one item for himself. Find a distribution of listed items that satisfies the above conditions.

Write numbers in the row titled distribution according to who gets the listed item: the father - 0, the eldest son - 1, the youngest son 2.

Item	briefcase	book	USB key	pen	exercisebook	pencil	marker
Price	30	20	14	10	9	6	1
Distribution	1	2	1	0	1	1	1

	Total
Eldest son	60
Youngest son	20

Evaluation	correct
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New

Hint

Instruction

Figure 1. The motivation problem

Students can write only into cells in the row that are shaded in blue. The sheet is initially saved with zeros in these cells. Additional data used to evaluate students' solutions are written with white font. For example, below the table total price of items received by each son is calculated using formulas with the logical function IF. Figure 1 shows one solution of the problem. The eldest son received items with a total price of 60 and youngest one in a total price of 20. The father kept the pen. The motivation problem has eight different solutions.

For smart students there is an open cell area at the bottom of the worksheet. Here they can enter other solutions to the problem by listing the items kept by the father and items received by the youngest son. If a student cannot solve the task, he can use the Hint button. Auxiliary information encourages students to consider whether the father can keep the briefcase or not. The student should be aware that the sum of the prices of all items is 90, so if the father kept the briefcase, the sum of the prices of the remaining items would be 60. Number 60 could help in solving the task, because it is divisible by 4. More detailed auxiliary information is provided by the Instruction button. The student is directed to think about those items the father cannot keep. For example, he cannot keep the book, because the total price of all the remaining items is 70. This total price cannot be divided between the sons so that the total price of all the items each one of them gets will be in a ratio of 3:1. In the following two tasks students should further investigate the divisibility by number 4 on concrete numbers. After that they have to create a two-digit number with one character "c" with a given property. Figure 2 shows an assignment of this kind together with one correct solution.

Create a two-digit number with one letter "c" to obtain the most two-digit numbers divisible by 4 after substituting a value for "c".

Tens	Units	Number of possibilities
c	2	5

New

Evaluation	correct
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Figure 2. Creating a notation of a number

An important task for the realizing the essential context is on the seventh workbook. The students' task is to create a three-digit number after replacing one digit with the letter "c". The task is then to

determine the most three-digit numbers divisible by 4 after substituting a value for "c". Figure 3 displays the task included in the assignment.

Create a three-digit number with one letter "c" to obtain the most three-digit numbers divisible by 4 after substituting a value for "c".

Hundreds	Tens	Units

Figure 3. Creating a three-digit number

If the letter "c" is in the units place, we can obtain at most three numbers divisible by 4. Students have already found that by placing letter "c" in the tens place in double-digit numbers; they can be obtain at most 5 numbers divisible by 4. The question is whether it would be the same with a three-digit number. Furthermore there is a question whether it would be appropriate to place the letter "c" in the hundreds place. Considerations leading to answering these questions should bring students to write a three-digit number using powers of 10, and to realize that any multiple of 100, and even higher power of 10, is always divisible by 4. From which there is very little is left to discover the divisibility rule for number 4.

3. Interactive learning activity about a cyclic quadrilateral

The previous learning activity is based on a sequence of tasks. This creates a linear learning path as characterized by B. F. Skinner, and is based on linear learning programs. Additional information and tasks allow creating alternative branches alongside the main branch. By including other branches we can create so called "branched programs", whose basic principles were elaborated by N. Crowder. The purpose of feedback in such programs is not only to evaluate student success, but also to guide the learning process.

The nature and ways of providing instrumental help-seeking can be the basis for the classification of learning systems. When designing interactive activities about cyclic quadrilateral we used the main ideas of the theory of contingent tutoring [2]:

- A system offers immediate feedback information about the success of the learning process, it provides advice and helps students to solve problems.
- Help should be graded in levels - four levels of help are recommended.
- The first three levels of help gradually clarify the solving of a given problem.
- Last level of help provides detailed explanation of the process of solving the problem.
- A system, in many cases, is not sufficiently sensible to recognize what kind of help a student needs and it does not provide the student with the required help. If the deficiencies are reflected in student's further learning activities too, the student needs to be given also unrequired help from the teacher.

We incorporated dynamic constructions into the system of auxiliary information. By doing so, students can experiment with geometric shapes and explore the relationships between them. To arouse the interest of students we promote problem-solving to introduce the activity. One task could be to investigate the conditions under which a parallelogram is circumscribed. Through dynamic construction students can investigate particular quadrilaterals that can be circumscribed. These could include squares and rectangles. After correctly solving this initial problem, we introduce the concept of a cyclic quadrilateral. The students are asked to investigate the properties of the angles of a cyclic quadrilateral (see Figure 4).

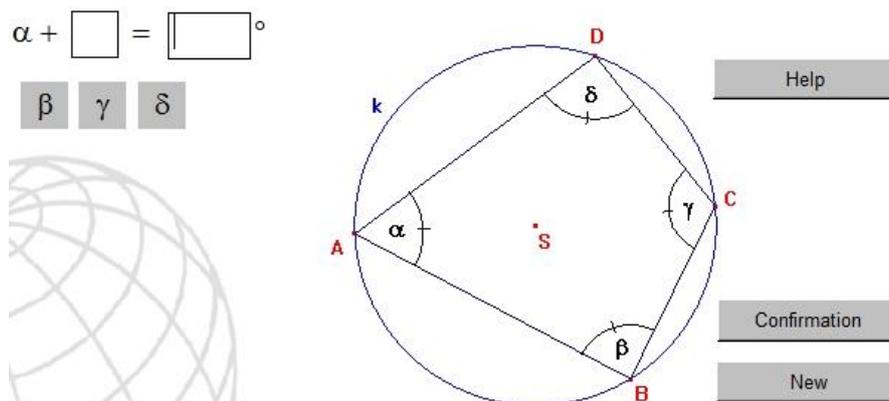


Figure 4. The main problem of the activity

If a student is able to find a relationship between the sizes of the opposite interior angles in a cyclic quadrilateral ABCD, a window summarizing the solution of the problem will be shown after entering the correct answer. Student will be explained the connection of the new knowledge to the solution of the initial problem situation. If necessary, the student can use the first level of help. After pressing the Help button, he is given a help in a new window to use the knowledge about central and inscribed angles. Student can also use another dynamic construction to see these angles in circles and to revision their properties.

1	2	3
<p>For the given pair of opposite interior angles in the cyclic quadrilateral ABCD apply the relationship between the inscribed and the corresponding central angle in the circle k.</p>	<p>Think about which inscribed angles formed by the same sides and diagonals of the cyclic quadrilateral ABCD have the same size and then based on the formula for the sum of interior angles of a quadrilateral try to find a formula for the size of interior angles at the opposite vertices of the quadrilateral ABCD.</p>	<p>Think about the properties of triangles with one vertex in the centre of the circle k and about the sizes of angles and these triangles. Try to find a formula for the size of interior angles at the opposite vertices of the quadrilateral ABCD based on the formula for the sum of interior angles of a quadrilateral.</p>
<p>Select the serial number of your chosen process and try to resolve the task.</p>		<p><input type="text"/> Confirmation</p>

Figure 5. Different ways to solve the main problem

After entering an incorrect or incomplete answer, the second level of help appears in another window. Student is indicated with possible ways his discovery can carry on. Advices on how to use the previous knowledge in three different ways of dealing with the problem are shown in Figure 5. In each branch the student is directed to a page where he can formulate his finding about the size of the internal opposite angles in a cyclic quadrilateral. If he enters an incorrect answer again, the main step in solving the problem is explained (see Figure 6).

Your answer is incorrect again.

The diagram shows the opposite angles α and γ at the vertices A and C respectively, and the reflex angle BSD. This angle is the central angle to the angle γ and therefore its size is 2γ . Find the central angle corresponding to the inscribed angle α and try to find the sum of angles $\alpha + \gamma$.

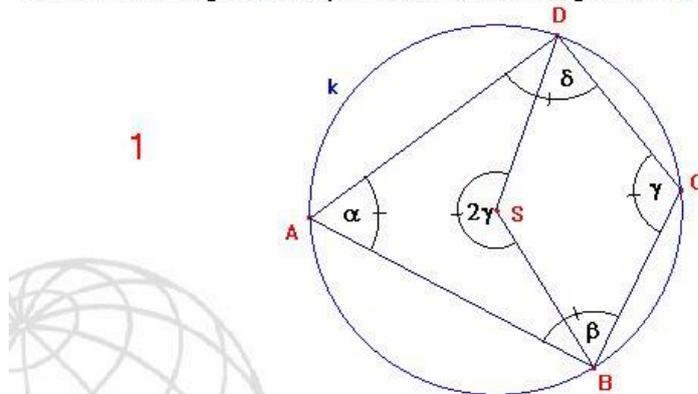


Figure 6. The main step in solving the problem in the first branch

If the third level of help does not lead the student to the correct solution, then the whole solution is explained to him. He must demonstrate an understanding of the explained relationships by solving an additional task. If the student can't solve this one he receives a warning to ask a teacher for help.

4. Conclusion

Described interactive activities were tested in school practice. Digital learning environment with properly administrated sequence of tasks led most students to the discovery of new relationships through individual learning pathways dependent on the degree of using auxiliary information. When working with a workbook to test divisibility by 4, some students did not know without the help of a teacher how to solve questions in which they had to use a letter "c" in the place-value notation of a number. Most of these students were not able to formulate the rule for divisibility by 4 on the last sheet of the workbook. Another lack was the vague formulation of discovered rule. For example, some students wrote a statement that a number is divisible by 4 if it is properly ended. When using the interactive learning activity Cyclic quadrilateral, auxiliary information provided by the system had been insufficient for a few students, so the system alerted them to seek help from the teacher. We also noticed that after using the first auxiliary information to explore the size of the opposite angles of a cyclic quadrilateral, a couple of students tried to find the answer based on their hand-drawn sketch. In this way they rather quickly got to the explanation of the solution to the problem without a deeper reflection.

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