THE "I HEART MATH" JOURNAL: A SELF-GUIDED TOUR OF PAPERT'S "MATHLAND"

by Sara Cole[1]

Abstract

By the time they reach secondary school, students often conclude that the study of mathematics consists mainly of memorizing formulas and procedures that a long-dead genius pulled from thin air at some point in the distant past. Who can blame them, when the mathematics that they see in school all too frequently does little to contradict those impressions? Modelled on Keri Smith's Wreck This Journal series[1], the "I Heart Math" journal is a collection of open-ended prompts that challenge students' deeply-ingrained beliefs about the nature of mathematics and about their own potential. The "I Heart Math" journal is an attempt to open the door to Papert's "Mathland," a world that is far removed from the worksheets and exercises that prepare students for success in many math classes [2]. Instead of offering a straight and narrow road, with the prompts in my math journal, I invite the journal's author to step off the beaten path to follow his or her interests and intuition in search of math that is surprising, intriguing, messy, and beautiful.

Keywords: school mathematics, journal, self-directed, student misconceptions, open-ended

1. Introduction

> Difficulty with school math is often the first step in an invasive intellectual process that leads us all to define ourselves as bundles of aptitudes and ineptitudes, as being "mathematical" or "not mathematical," "artistic" or "not artistic," "musical" or "not musical," "profound" or "superficial," "intelligent" or "dumb." Thus deficiency becomes identity and learning is transformed from the early child's free exploration of the world to a chore beset by insecurities and self-imposed restrictions. [2:p.8]

I began this project[2] as a photographer begins in a darkroom, with the negative image of my final product. I hoped that the finished product would be everything that, for me, a typical math class is not: creative, surprising, interconnected, personally meaningful, hands-on, and fun. Where a traditional math problem often defines every aspect of the question and proceeds along a linear set of steps to a single solution, I hoped to inspire students to define their own questions and then pursue them in unorthodox ways, to explore and play with pieces of math that strike their fancy, to doodle,

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[1] Harvard Graduate School of Education; sara_cole@mail.harvard.edu.

[2] Many of the ideas in this paper stem from my coursework at Harvard and from my teaching practicum at the Edwards Middle School. I owe a debt of gratitude to the students and teachers who inspired and refined these idea including: Professor Karen Brennan, Professor Robert Martinez, Professor Jon Star, Carla McCormack, Wilhelmina Peragine, Jimmy Kim, Emily Ruff, Marissa Martin, Amanda Ackiss, Emilie Stump, and Ben Gutzler.

to fiddle, to tinker. Instead of defining success as a single, correct *answer*, the "I Heart Math" journal defines success first as the asking of a good *question*. Instead of dividing mathematics into algebra and geometry and then continuing to subdivide into ever smaller units of definitions to memorize and ever narrower tasks to complete, the journal prompts hint at undiscovered connections in the mathematics you already know, and unseen layers all around you. Like the philosopher in *The Hitchhiker's Guide to the Galaxy* who cries out, "we demand rigidly defined areas of doubt and uncertainty!" [3:p.172] a math curriculum that creates divisions rather than connections seems ludicrously artificial when compared to the rich, interconnected world that the mathematician and the math lover inhabit. The "I Heart Math" journal is an attempt to open the door to that world, a place Papert calls "Mathland," to lay out the welcome mat and put on the kettle [2]. Instead of offering a straight and narrow road, with the prompts in my math journal, I invite the journal's author, the math student, to step off the beaten path to follow his or her interests and intuition in search of math that is surprising, intriguing, messy, and beautiful.

2. What's Wrong with Math in School?

What are the features of a typical math class that cause so many students (and adults) to grimace in distaste at the mere mention of mathematics? Schoenfeld [4] describes how the repeated transfer of formulaic, fragmented knowledge leaves a student with the impression that mathematics is nothing more than a collection of arbitrary, unrelated formulas. When math class follows this model, "many students come to believe that school mathematics consists of mastering formal procedures that are completely divorced from real life, from discovery, and from problem solving [4:p.197]." Moreover, students become passive receivers rather than active participants and problem solvers. According to Schoenfeld, "students who believe that mathematical understanding is simply beyond ordinary mortals like themselves become passive consumers of mathematics, accepting and memorizing what is handed to them without attempting to make sense of it on their own [4:p.198]." Each piece of mathematics becomes what Whitehead calls, an "inert idea" that is "merely received into the mind without being utilised, or tested, or thrown into fresh combinations [5:pp.1-2]." In sum, the traditional math class often transmits inert ideas to passive learners in formulaic and disconnected pieces.

Are there ways in which math instruction departs from the theory of student as empty vessel waiting for the teacher to transfer knowledge? Mathematics in school is, after all, supposedly a cumulative endeavor, a series of new tasks that rely on the collection of skills and knowledge that preceded them. Ironically, while teachers espouse the rhetoric of creative problem solving often enough that students claim to believe that "one can be creative in mathematics," those same students report that "mathematics is best learned by memorization [6:pp.348-9]." Perhaps teachers oversimplify rich, complex ideas and create artificial boundaries in a well-meaning effort to avoid confusion and struggle on the part of the student. Or perhaps they streamline to save time in an overstuffed curriculum and to prepare students for high-stakes tests [6-8]. Unfortunately, that simplification can reduce a rich question to an exercise in rote memorization, a one-way transfer of facts and procedures from teacher to student.

3. Why a Math Journal?

The "I Heart Math" journal sprang from a desire to create a space where the passive consumer of mathematics can become an active creator without fear of embarassment or failure. In secondary and post-secondary mathematics classrooms, journal writing has proven to be beneficial for both the emotional and cognitive well-being of students [9-12]. In particular, Borasi & Rose concluded that if student struggles in math "appear to stem from preoccupation with product and answers, anxiety, external manipulation of symbols, and passive and disengaged learning, then our analysis suggests that journal writing may provide a valuable addition to current modes of mathematics instruction [12:p.362]." Papert's "object-to-think-with" is a meeting place for personal interest, cultural relevance, and new knowledge [2]. The act of creating a math journal transforms the student from audience member to author, from passive consumer to active participant.

The "I Heart Math" journal is a mathematical object-to-think-with, a place where students can reap the benefits of "externalization and articulation [13:p.12]" with no fear of failure. Papert reminds us that some divisions run deep into our collective psyche, and I find that many of my students self-label as "not mathematical." Dweck's work [14] confirms the "insecurities and self-imposed restrictions" that Papert describes. She found that some students, for example, so internalized their failures as a reflection on their abilities that they predicted they would be unable to solve a new problem, in spite of the fact that the new problem was in fact one they *had already solved correctly* before they experienced failure. A journal is a collection of ideas and opinions instead of correct or incorrect answers. It is a safe space for math students who disengage as a result of past failures.

3. What Are Student Misconceptions About Mathematics?

In designing the "I Heart Math" journal, I planned to build on the student's knowledge of math facts and concepts, but I understood that at the same time I would have to address another kind of prior knowledge. Returning to Papert's words, it was clear that students would bring with them not only the knowledge of definitions and formulas, but also a set of beliefs about their own deficiencies and strengths, and about the nature of mathematics. To return to the "child's free exploration" that Papert described, I would first have to dismantle a lifetime of "self-imposed restrictions." Smith, diSessa, and Roschelle [15] question the assertion that all student misconceptions are impediments to learning. They argue that misconceptions can be productive elements in the student's evolving "knowledge systems [15:p.117]." I chose to focus on addressing three key misconceptions which I believe to be unproductive. I believe that these misconceptions cause students to devalue certain approaches to mathematics and that the experience of doing mathematics would be richer and more engaging if it included these approaches. In continuing, I will consider each of these misconceptions and the ways in which they shaped my project.

3.1 Mathematics is Logical and Calculating Rather than Intuitive and Creative

Let us first consider the idea that mathematics should be abstract, formal, and impersonal. Rather than intuitive and creative. Where does this misconception come from? Turkle and Papert [16] discuss the privileging of abstract, formal reasoning over hands-on, improvisational thought processes. As an example, they point to Piaget's portrayal of concrete reasoning as a developmental stepping stone on the way to superior, abstract reasoning skills. Likewise, Lévi-Strauss relegated the concrete work of "bricolage" to those unable to reason in more abstract ways [16]. In elementary math classrooms and post-secondary computer programming courses alike, Turkle and Papert found anecdotal evidence of a preference for abstract, formal reasoning over creative, unorthodox, and concrete approaches.

Their evidence also suggests, however, that students who struggle with abstract, formal, impersonal reasoning are far from doomed to fail in a math class. If we follow Turkle and Papert's advice and validate improvisational, highly personal approaches, those students may be able to demonstrate skill and ability in mathematics. Like the bricoleur whose final product grows organically out of a "negotiation" with the raw materials, students could play with mathematics [16:p.138]. Instead of memorizing formulas only to forget them as soon as the test ends, they may discover areas of mathematics that interest them and pursue those interests through experimentation and exploration.

3.2 Mathematics is Predictable and Linear Rather than Messy and Surprising

Many students believe that mathematics is linear, that it progresses from a well-defined problem through a single, correct method of solution to a single, correct answer. The clean, organized mathematics that students often see in class only serves to reinforce their belief that mathematical thought should be organized and linear [4]. When the math they see is so often sanitized and streamlined, it is only natural for the students to "imagine that some minds are tidy and neat and sharp [17:p.15]" and to, in turn, hide their own messy thoughts.

In reality, math is anything but linear. As Schoenfeld [4] illustrates, expert mathematicians test and discard many solutions before finding one that works, and there are many different ways to successfully reach a solution. If students worked to solve questions that had no right or wrong answers in a space where creative messiness and novel, unexpected thought was not only permitted but celebrated, they might see the merit in their own messy thoughts.

3.3 Mathematics is Fragmented and Disjointed Rather than Interconnected

While I aim to see students thinking about math in non-linear, "messy" ways, I also want to see them making sense of the mess, I want them to see that math is interconnected not disjointed. When students make connections where before there were only divisions, the disjoint topics of classroom mathematics gain meaning and context, and Papert's Mathland begins to appear. Skemp offers "sets, mappings, and equivalencies" as examples of common themes that run through supposedly disconnected branches of mathematics, and argues that much of the power of these themes is lost when teachers present them as "separate topics, rather than as fundamental concepts by which whole areas of mathematics can be inter-related[8:p.159]."

4. A Journal to Dismantle These Misconceptions

Keri Smith published a book called *Wreck This Journal* in 2007, and the "I Heart Math" journal is modelled on her work [1]. Smith's journal has a subversive, rule-breaking, boundary-crossing quality. Smith uses open-ended prompts to help the journal author break free of preconceived notions of what a journal is, what a book is, and how a person should interact with those things. Ideas about what math is and how we should interact with math are quite similar to these ideas about books and journals. Both sets of beliefs are so deeply-rooted that we rarely question them, and may not even be consciously aware of them. The goal of my journal is to push students to recognize and re-consider long-held, limiting misconceptions about math by giving them the freedom to play with math in non-linear, unorthodox ways.

Consider, for example, the practice problem and solution in Figure 1. The student who completes this problem is learning to execute the binomial theorem as a pre-determined sequence of formulas and operations that must be carried out without error to successfully reach the answer. Instead of exploring the patterns that emerge from the distributive property to form the binomial theorem, the student is busy looking for a calculator to figure out what 42^3 equals.

1.) $(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^k a^{n-k}$

$$(x+42)^3 = ?$$

$$(x+42)^3 = \sum_{k=0}^{3} \binom{3}{k} x^k 42^{3-k}$$

$$(x+42)^3 = \binom{3}{0} x^0 42^{3-0} + \binom{3}{1} x^1 42^{3-1} + \binom{3}{2} x^2 42^{3-2} + \binom{3}{3} x^3 42^{3-3}$$

$$(x+42)^3 = 42^3 + (3x)(42^2) + (3x^2)(42) + x^3$$

$$(x+42)^3 = 74{,}088 + 5{,}292x + 126x^2 + x^3$$

Figure 1 - A standard practice problem.

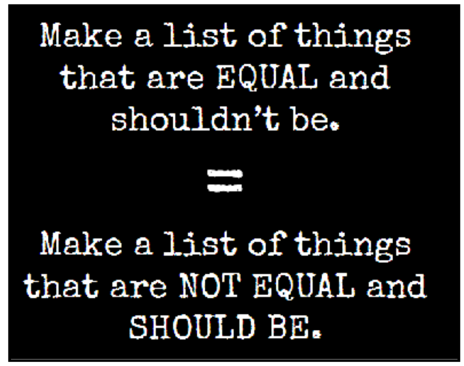

Figure 2 – A journal prompt.

To answer a question in the "I Heart Math" journal, on the other hand, the student must first decide what the question means. In that first step, and in the work that follows, the journal pushes the student to seek out the unexpected. Instead of employing the property of equality to solve an equation or manipulate an expression, the journal prompt in Figure 2, asks the student to question the nature of equality. It asks the student to look for instances that seem to defy the patterns and rules of mathematics and to search for rules that are counterintuitive.

Students believe that mathematics is formal, abstract, and impersonal. The journal challenges them to find mathematics that sparks their interest and pursue it through the use of intuition and creativity. Students strive for a smooth progression from a well-defined problem through a single correct method of solution to a single right answer. The journal values novelty and surprise over consistency and uniformity, the asking of interesting questions over the production of correct answers. Students believe that mathematics is handed down from on high in disjointed fragments. The journal asks students to build new knowledge by finding connections in the mathematics they already know.

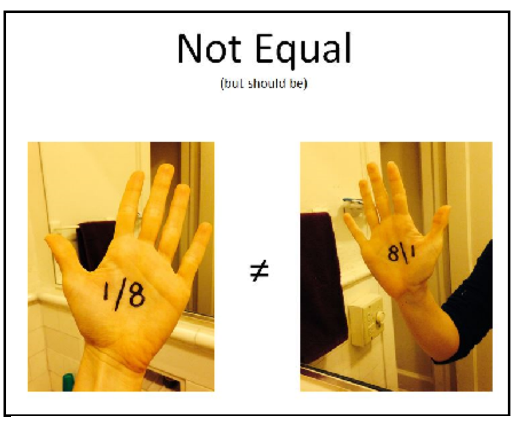

Figure 3 – Response to journal prompt. Reprinted with the author's permission.

$\frac{\frac{3}{4}}{\frac{3}{4}} \neq 3 \div 4 \div 3 \div 4$	$\frac{42}{42} = \frac{\frac{7}{7}}{\frac{3}{3}}$
$0 \times \infty \neq 0$	$\infty + 1 = \infty$
$3{,}000 \neq 3$	$3.000 = 3$
$1 \neq 0^0 \neq 0^1 \neq 0$	$1^0 = 1^1 = 1$

Figure 4 – Response to journal prompt.

In the journal's final revisions, I struggled to balance the desire for freedom and play with the need to elicit complex and interconnected mathematics. Multiple authors identify the nature and specificity of journal prompts as a key factor in producing high quality responses from students, and in the journal's ability to influence student beliefs and thought processes [10, 12, 18]. The final version of the journal prompts attempts to impose productive constraints without limiting the author's freedom to improvise. The variety of responses (Figures 3 and 4) that this single prompt inspired is evidence that it is possible to ask open-ended questions that provide both freedom and rigor. To view the evolving collection of journal prompts and responses, please feel free to explore the google presentation at: http://bit.ly/1i6WWHz.

5. Conclusion

Looking ahead, I plan to share the "I Heart Math" journal with secondary school students, and to continue refining and creating journal prompts based on their feedback. While a single journal cannot reverse a lifetime of self-doubt and negative encounters with mathematics, I am optimistic that the "I Heart Math" journal can shift the ways that students think about mathematics. Through their journals, mathematics lovers and haters alike can venture into uncharted territory, guided by their own intuition and interests. Wandering around in Mathland, they might just decide to take off their coats, pull up a chair, and stay awhile.

References

[1] K. Smith, *Wreck This Journal: To Creat is to Destroy*: Penguin Group (USA) Incorporated, 2012.

[2] S. Papert, *Mindstorms : children, computers, and powerful ideas*. New York: Basic Books, 1980.

[3] D. Adams, *The Hitchhiker's Guide to the Galaxy*: Random House Publishing Group, 2005.

[4] A. H. Schoenfeld, *Cognitive science and mathematics education*. Hillsdale, N.J.: Lawrence Erlbaum Associates, 1987.

[5] A. N. Whitehead, *The aims of education & other essays*. New York: The Macmillan company, 1929.

[6] A. H. Schoenfeld, "Explorations of Students' Mathematical Beliefs and Behavior," *Journal for Research in Mathematics Education,* vol. 20, pp. 338-55, 07/01/ 1989.

[7] M. Suri. (2013, November 21, 2013). How to fall in love with math. *The New York Times*. Available: http://www.nytimes.com/2013/09/16/opinion/how-to-fall-in-love-with-math.html?_r=1&

[8] R. R. Skemp, *The psychology of learning mathematics*: L. Erlbaum Associates, 1987.

[9] K. P. Chapman, "Journals: Pathways to Thinking in Second-Year Algebra," *Mathematics Teacher,* vol. 89, pp. 588-90, 10/01/ 1996.

[10] H. P. Koirala, "Facilitating Student Learning through Math Journals," in *Annual Meeting of the International Group for the Psychology of Mathematics Education*, Norwich, England, 2002.

[11] B. J. Dougherty, "The Write Way: A Look at Journal Writing in First-Year Algebra," *Mathematics Teacher,* vol. 89, pp. 556-60, 10/01/ 1996.

[12] R. Borasi and B. J. Rose, "Journal Writing and Mathematics Instruction," *Educational Studies in Mathematics,* vol. 20, pp. 347-65, 01/01/ 1989.

[13] R. K. Sawyer, *The Cambridge handbook of the learning sciences*. Cambridge ;New York: Cambridge University Press, 2006.

[14] C. S. Dweck, *Self-theories : their role in motivation, personality, and development*. Philadelphia, PA: Psychology Press, 2000.

[15] J. P. Smith Iii, A. A. diSessa, and J. Roschelle, "Misconceptions Reconceived: A Constructivist Analysis of Knowledge in Transition," *Journal of the Learning Sciences,* vol. 3, p. 115, 1993.

[16] S. Turkle and S. Papert, "Epistemological pluralism: Styles and voices within the computer culture," *Signs: Journal of Women in Culture & Society,* vol. 16, p. 128, 1990.

[17] S. Papert, "A word for learning," in *Constructionism in practice : designing, thinking, and learning in a digital world*, Y. B. Kafai and M. Resnick, Eds., ed Mahwah, N.J.: Lawrence Erlbaum Associates, 1996, pp. 9-24.

[18] E. L. Mann, "Creativity: The Essence of Mathematics," *Journal for the Education of the Gifted,* vol. 30, pp. 236-260, 12/01/ 2006.