FIBONACCI FOREVER: CLASSICAL MATHEMATICS AND GEOMETRY FOR DESIGN.

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Abstract

Writing about architecture over two millennia ago, Vitruvius listed the ideals of strength, usefulness and beauty in buildings and constructions. In mathematics also, there are certain topics based on mathematics from antiquity that arguably have the same characteristics of firmitas, utilitas and venustas, and these topics may allow a window of opportunity for inspiring creativity in both students and teachers alike. This is an account of the integration of classical mathematical applications within a community college mathematics course, Geometry for Design. For comparison, outcomes for students studying Geometry for Design course were contrasted with outcomes for students studying Intermediate Algebra, a similar-level mathematics course. Geometry for Design students had a better chance of success --obtaining an A,B or C grade-- than students who studied Intermediate Algebra, while students who studied Geometry for Design had about the same rate of persistence compared to students who studied Intermediate Algebra.

Keywords geometry, algebra, success, community college

Introduction

This is an account of the integration of classical mathematical applications within a community college mathematics course, MATH 137 Geometry for Design. MATH 137 is an interdisciplinary, technology–enhanced geometry course that was originally designed to serve students in the College's Architecture, Design and Construction (ADC) program. However, students from other curricula can also study MATH 137. The incorporation of a number of topics from classical mathematics can create an interesting and challenging environment for learning and doing mathematics. For comparison, outcomes for students studying Geometry for Design course were contrasted with outcomes for students studying MATH 118 Intermediate Algebra, a similar-level mathematics course. MATH 118 is a traditional, lecture-based course with no designated classroom technology component, and it is usually the terminal college-level mathematics course taken by the majority of students at the Community College of Philadelphia. Outcomes for students in four sections of Geometry for Design courses were compared with outcomes for students in eight sections Intermediate Algebra courses which took place during the academic years 2012 and 2013. Geometry for Design students seem to have a higher rate of success -- obtaining an A,B or C grade -- than students studying Intermediate Algebra. Students in both courses seemed to have about the same rate of persistence.

Community College of Philadelphia and the ADC Curriculum

Community College of Philadelphia is an open admission institution that offers associate's degree and certificate programs in the liberal arts and sciences and career technologies. Its mission is to provide a coherent foundation for college transfer, employment and life- long learning. The College serves a diverse, urban community, with students from a wide range of skills, backgrounds and ages, and the challenge is to help students acquire basic academic skills that will enable them to achieve their goals. The College was created in 1964 and is part of the Pennsylvania State system of two-year colleges. Currently, there are more than 15,000 FTEs enrolled in courses, and, since its inception, the College has served more than 685,000 students. The College has a Center City Main Campus, with three other satellite campuses located in the North-East, North-West and West regions of the city. Students can enroll in classroom or online courses, and, in addition to academic degree and certificate programs, many non-credit offerings are available. The Architecture, Design and Construction (ADC) curriculum offers students several pathways to successful careers in Computer Assisted Design, Interior Design, Construction Management, Energy Conservation, Geographic Information Systems and Architecture.

MATH 137 Geometry for Design

In 1998, faculty from the Architecture and Mathematics Departments collaborated to produce a new course, MATH 137 Geometry for Design. This project was supported by funding from the U. S. National Science Foundation and the course was originally designed to serve students in visual design curricula, such as construction technology, computer assisted design, art and interior design. MATH 137 is now a required course for ADC students, and for the last sixteen years, a fall and spring section of the course has been offered. Recent semesters are showing increased demand for the course, which now runs an additional section delivered during a seven-week summer semester. MATH 137 Geometry for Design classes have always been held in the College's standard computer-equipped classrooms and regular classrooms, but now all sessions are held in the state-of the art classrooms of the ADC Department .

As author Michael Serra [1] was designing his unique text, *Discovering geometry: an investigative approach* , he was guided by the research of Pierre and Dina Van Hiele concerning how students learn geometry. Serra chose to introduce geometric ideas visually, and then, via analysis and induction, to lead up eventually to deduction and proof. It is, arguably, a quintessentially constructionist approach to learning geometry, with students observing, measuring, creating definitions, and honing their powers of reasoning through doing mathematics, not merely following a routine "cook-book" approach. Likewise, our experience suggested that incoming Community College of Philadelphia students often have very little understanding of geometrical ideas, yet they are expected to be able to deal with sophisticated two- and three-dimensional geometric concepts throughout their ADC studies. Wayne Nirode [2] laments that “Although the advent of dynamic geometry software (DGS) saved geometry from the “trashcan of history”, the high school geometry course is still dangerously close to being thrown away”. Our students’ lack of geometrical knowledge may be related to recent educational initiatives, such as the Common Core (CCSSI 2010), which places a far greater emphasis on learning algebra. Accordingly, Community College of Philadelphia course developers agreed that, in contrast to other branches of mathematics that rely heavily on abstraction, the study of geometry should begin with first-hand experiences and observations from the physical world. Therefore, course writers stressed experience-based learning, and so many of the lecture lesson-plans include hands-on activities and computer-based inquiry. MATH 137 Geometry for Design combines a sound preparation in the basic concepts and techniques of plane and solid geometry with exploration of the concrete applications of geometry in fields such as architecture, construction, art and design. With the strength, usefulness and beauty prized by Vitruvius, topics from classical mathematics are naturally included in the course. The MATH 137 syllabus includes traditional straightedge-and-compass methods of geometric construction, properties of triangles, polygons and circles, plane transformations, symmetry and tessellations of two-dimensional figures, area of geometric figures, properties of 3-dimensional polyhedra, volume of solids, the Pythagorean Theorem, and ratio and proportion and similarity of figures and objects. Although the main mode of reasoning is inductive, deductive reasoning and concomitant study of formal proof are included. As the course has evolved over the years, topics from art and architecture, such as perspective drawing and building design, have been incorporated into the course. Students research arches and domes and the course concludes with an exploration of the Golden Mean and its relevance to architecture, art, science and design. Students create portfolios of manual and computer-based drawings and build 3-dimensional models. Geometer’s Sketchpad software is used extensively to explore ideas, make conjectures and literally draw conclusions and formulate proofs. Sessions are held in computer-equipped classrooms, and internet web sites relating to geometry, architecture, construction, art and design are frequently accessed. Student achievement is assessed by computational homework quizzes, midterm and final exams, two short written papers and a portfolio of computer-generated drawings.

Hands-on, computer-based activities with classical mathematics

To assist with building knowledge structures and strengthening problem-solving skills, students in the MATH 137 Geometry for Design course explore classical mathematics via applications that use computer technology and hands-on activities. First explorations feature three-dimensional puzzles with multiple solutions requiring good visualization skills, and these are helpful for introducing students to working co-operatively. Students are exposed to inductive thinking and formula development. For example, using inductive reasoning, they are tasked to discover a formula that computes the number of diagonals of a daunting-looking 18-gon [3]. It is clear that ADC students find that strong drawing and visualization skills are indispensable in many real-world situations in their professions. The use of Geometer's Sketchpad software is introduced at the beginning of the course, and students with a background in CAD (computer assisted design) or art are usually adept at using technology to create diagrams and drawings. The skills of observing, measuring and drawing are also indispensable. Students are required to master traditional classical compass-and- straightedge techniques of construction, and they also can use Geometer's Sketchpad to render computer graphics-based incarnations of classical constructions. As their predecessors have done for centuries, students use traditional drawing tools to create the four points of concurrency of a triangle (incenter, circumcenter, orthocenter, and centroid). However, unlike their earlier counterparts, they can also use computer-based technology to meaningfully explore topics from classical mathematics.

The constant π is one of the most important topics of classical and contemporary mathematics. Students use Sketchpad to draw circles and use its onboard calculator to try to discover a formula relating the circumference and radius (or diameter) of the circle. Dividing the easily measured circumference by the circle's radius yields 6.28, approximately twice the value of π, and after some thought, most students will recall this number from previous high-school math courses. Other activities include rolling out the circumference of a wheeled object (e.g. a trolley or a bicycle tire), measuring its diameter, and thus discovering an approximate value for π. ADC students, some of whom wish to pursue careers as professional surveyors, can thus better understand the mathematics behind some real-world applications, such as the odometer and pedometer.

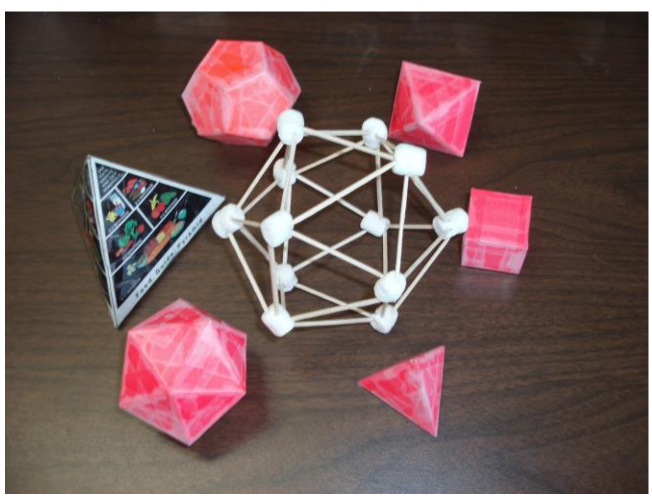
Another topic which draws its inspiration from classical mathematics is the derivation of the area of a circle, πr^2. This was first deduced by, most likely, Eudoxus [4] , using the "method of exhaustion". Using similar reasoning, by juxtaposing 12 sectors of a circle into a “rectangle” , the circle's area can be suggested by approximating the area of the "rectangle" as (radius) x (½ the circumference), or $r \times 2\pi r/2$, hence πr^2 .

Students learn about translation, reflection and rotation using hands-on activities that are greatly facilitated by the use of Geometer's Sketchpad. It is best to demonstrate these isometries by first drawing a non-symmetric polygonal shape, such as a familiar alphabet letter like a capital F. Students are challenged to discover how to first reflect the object they have created using a mirror line, then to rotate the object, and finally to translate their object by a marked vector. ADC students quickly recognize that the same transformations are routinely used in the design software applications that they employ in the drafting of buildings and their interiors. Using Sketchpad, the next step is to compose transformations such as two reflections in parallel mirror lines (equivalent to a translation), and then to compare two reflections in intersecting mirror lines (equivalent to a rotation). Tessellations or tilings are frequently encountered in the worlds of architecture and design. Students use Geometer's Sketchpad to explore their creativity as they construct their own tessellations using translations and rotations. In addition , they view web sites and videos of the work of visual artists such as M.C. Escher, who was strongly inspired by geometry in particular.

While exploring one of classical mathematics' most important topics, the Theorem of Pythagoras , students use marked strings, or "fairy lights", recalling knotted ropes used by Ancient Egyptians, to create large size right-angled triangles in space. ADC students can relate this classroom activity to modern-day laser-based construction site procedures, for example in establishing perpendicular corners for the foundations of buildings. Proofs of Pythagoras's Theorem are presented, including those by Bhaskara, Leonardo da Vinci and U.S. President James Garfield. Students use Geometer's Sketchpad to explore a hands-on "juxtaposing areas" proof. The related hands-on problem of anchoring a boat in a stream or river is also used to help students visualize the importance of the right-angled triangle theorem and its applications.

Many students confuse area and volume, so it is very important to help them visualize the concept and hence understand formulas for volumes of geometric solids. Math manipulatives such as $1cm^3$ blocks are used to construct rectangular solids. Students use hollow translucent plastic solids that can be filled with water to demonstrate that the volume of a cone is one third the volume of a cylinder of the same radius and height. The formula for the volume of square and triangular pyramids is inferred in similar fashion. Thus, students seem to better understand these indispensible volume formulae.

The subject of the Platonic Solids is a famous topic from classical mathematics, and students use nets to construct cardboard models of the five Platonic Solids. Using these examples, students build models of polyhedra using toothpicks (edges) and mini-marshmallows (vertices) to discover the Euler formula, $V + F - E = 2$, [3]. Students learn about complex polyhedra in the contexts of art, architecture and art history by exploring the encyclopedic web site "georgehart.com " [5].



When studying congruency and similarity, students create compass and straightedge geometric constructions. They are further engaged by computer-generated activities to investigate congruency and similarity criteria for triangles and polygons. The list of topics from classical mathematics would probably be incomplete without including the Fibonacci Sequence and the Golden Section, Phi. There are literally millions of web sites featuring these important topics, which have implications for biology, art, architecture and design. Students construct Golden rectangles using compass and straightedge techniques, and students observe real word objects that are designed as golden rectangles, such as credit and ID cards, cigarette packs, and the classic iPod. Students discover that the name of their home town, Philadelphia, contains a famous Classical mathematical constant, Phi.

Intermediate Algebra MATH 118

At the Community College of Philadelphia, entering students are required to take a mathematics placement test and approximately 75% are assigned to Arithmetic, Elementary Algebra or MATH 118 Intermediate Algebra courses. Currently, MATH 118 is a prerequisite for most degree or certificate programs at the College. Most students taking engineering, business, and science and technology curricula study MATH 118. Topics include the Real number system, systems of linear equations and inequalities, polynomials, rational expressions, radical expressions, and quadratic equations. It is challenging to introduce hands-on, Classical mathematics-based material into this course, as it is fast-paced and more abstract than MATH 137 Geometry for Design. Most MATH 118 sections are taught in classrooms using a lecture-based format, with no access to classroom computer use. Most instructors do not permit the use of calculators in class, whereas students expect to be able to use calculators, computers and even cell phones for graphing and computation. The assigned textbook [6] is accompanied by Web-based MyLabsPlus software which is used as a supplement for tutorial and practice. Since Spring 2010, MyLabsPlus has also been used to administer a departmental final exam. Student achievement in the MATH 118 sections considered in this small comparison study is measured by computational homework quizzes, a midterm and a departmental final exam.

Comparison of outcomes for Geometry for Design and Intermediate Algebra

To compare the achievement of students who studied MATH 137 with the achievement of students who studied MATH 118, recent records of students' final grades were analyzed.

Retention for students in each set of courses was also compared. Data was obtained from four sections of Math 137 and eight sections of Math 118 that were taught by the author during two academic years, from Spring 2012 through Fall 2013. "Success" was defined as earning an A, B or C grade . For these MATH 137 students, the success rate was 67.8 %, while for these MATH 118 students the success rate was 58.7%. This difference in sample proportions was significant at the 5% level. Therefore students who studied MATH137 Geometry for Design may have an increased chance of obtaining an A, B or C grade, compared to students who studied MATH 118 Intermediate Algebra. In addition, statistics from the Office of Institutional Research at the Community College of Philadelphia indicate that the over-all rate of success for the entire cohort of MATH 118 from Spring 2012 to Fall 2013 was approximately 45%. Although the Geometry for Design population is very small (hundreds) compared to the overall population of Intermediate Algebra (thousands), the Geometry for Design students' rates of success seem to be significantly greater. This result is in line with data from earlier semesters, when the difference in rates of success for the two courses was again significant. The author's earlier surveys of students' attitudes with regard to learning Geometry for Design suggest that students felt that the material was interesting and useful and pertained to their career areas. This reaction was not usually encountered with students who were studying Intermediate Algebra. It is possible that a hands-on, constructionist approach and the wealth of interesting and useful topics from classical mathematics may have had some influence on success rates for students. With regard to retention, the drop rate for Geometry for Design students was compared with the drop rate for Intermediate Algebra students. For the four sections of Geometry for Design, the students' average drop rate was approximately 20% , compared to the eight sections of Intermediate Algebra, whose students had a drop rate of approximately 20%. The different approaches used for teaching and learning seemed to have no effect on retention. Further study to investigate these different rates of success is indicated.

Conclusion

Data indicate that Geometry for Design students succeed at a higher rate than students who studied Intermediate Algebra. Further study is required before drawing any broader conclusions. Applications from classical mathematics pertaining to the contexts of construction, architecture and design appear to interest and motivate students as they learn some useful, powerful and beautiful mathematics. This, in turn, may translate into higher success rates. Future initiatives might investigate the increased incorporation of topics from classical mathematics into other geometry and algebra courses.

References

[1] M. Serra, *Discovering geometry: an investigative approach*, 4th ed. Emeryville CA, Key Curriculum Press, 2008.

[2] W. Nirode, “Don’t sacrifice geometry on the Common Core altar.” , *Mathematics Teacher*, vol.107, no.3, pp.168-170, October 2013.

[3] D. Seymour and E. Beardslee, *Critical thinking activities in patterns, imagery and logic*. Dale Seymour Publications, USA, 1990.

[4] C. Boyer, *A history of mathematics*, 2nd ed. New York, NY: John Wiley & Sons, 1991.

[5] G. Hart, (2014, Apr .) *George W. Hart*, Available: http://www.georgehart.com

[6] E. Martin-Gay, *Intermediate Algebra*, 6th ed. Upper Saddle River, NJ, Pearson Prentice-Hall, 2013.