

# ALGEBRAIC EXPRESSION MACHINE: A WEB AD HOC LEARNING ENVIRONMENT FOR DEVELOPING STRUCTURE SENSE

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## **Abstract**

*We designed and developed an interactive web learning environment that resulted in a task based algebraic machine. This learning environment was designed ad hoc [1] to develop structure sense in algebra in high-school students. This is the special purpose of the machine we developed.*

*In [2], Hoch and Dreyfus define the term structure sense as a list of advanced skills involving recognizing known structures in a complex algebraic expression and taking advantage of that recognition in order to be efficient in algebraic manipulation.*

*We believe that algebraic substitution is the basis for developing structure sense and that this interactive Algebraic Expression Machine will support students to learn and reinforce substitution and make them think about the structure of algebraic expressions.*

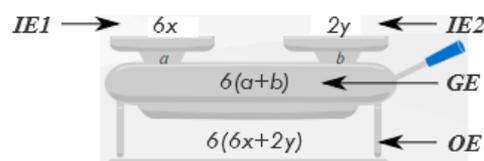
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## **1. Algebraic Expression Machine**

The ad hoc development was designed according to the scheme of a machine or system (used in the design of artifacts) that consists of, in its simplest form, *input*, *process* and *output*.

In the case of this algebraic expression machine, these three components are (see Figure 1): two algebraic expressions as input (named IE1 and IE2); a generating expression (GE) as process; and an expression that results from replacing IE1 and IE2 in GE (output expression OE).

For example, if IE1 is  $9x$  and IE2 is  $6y$ , and the GE is  $6(a + b)$  then the machine will return the OE  $6(9x + 6y)$  because it substitutes  $a$  by  $9x$  and  $b$  by  $6y$  (Figure 1).



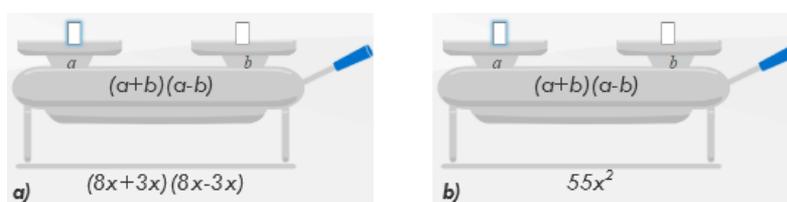
**Figure 1** Components of the expression machine: IE1 and IE2, input expressions; GE, generating expression; OE, output expression

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According to its structure, the machine allows 3 different tasks organized as scenarios (inspired in simple mobile video games): given two of these components, students are asked to find the missing one. So there are 3 different scenarios: *Find Output*, *Find Input* and *Find Generating Expression*.

Each scenario has a series of increasing difficulty exercises (levels). Students must type algebraic expression(s) in text field(s) of missing component(s) to complete the level, depending on which scenario they are in. By running the machine the student can observe if her algebraic expression(s) match the expected result.

The structure of the given expressions changes with the levels of each scenario to increase the difficulty of the exercises. Consider, for example, Scenario *Find Input* and Figure 2. In this figure, two different exercises of this scenario present the same GE  $(a + b)(a - b)$  and OE's that are equivalent, although different in structure. OE (Figure 2-a) is  $(8x + 3x)(8x - 3x)$  and OE (Figure 2-b) is  $55x^2$ .



**Figure 2** Two exercises with the same GE and equivalent OE's, but different in structure, have different levels of difficulty

In the first case a solution arises almost immediately since here OE and GE have the same structure. Thus the solution  $a = 8x$  and  $b = 3x$  comes naturally. In contrast, in the second exercise, GE and OE ( $55x^2$ ) have different structures: one is a product of binomial conjugates and the other a monomial. This contrast of structures makes it more difficult to find two input expressions as a possible solution and will need a thoughtful work with paper and pencil algebra. Note that exercises in scenario *Find Input* have no unique solution.

In advanced levels of this learning environment, the students must construct in paper and pencil the machine missing expression(s) according to the restrictions imposed by the activity. In other words, it's very unlikely that a trial and error approach leads the student to complete the given tasks.

### 3. Final remarks

We think that algebraic substitution is the basis for developing structure sense and that the designed levels in each scenario will support students to learn and reinforce substitution and make them think about the structure of algebraic expressions. We are about to test this ad hoc development with high school students. Some tests were conducted with experts. This work was supported by CONACyT Ref. 168620 and 316578. The working demo can be found at [ae-machine.matetam.com](http://ae-machine.matetam.com).

### References

- [1] T. Rojano, "Recursos multimedia y el libro de texto gratuito: entre las herramientas universales y los desarrollos ad-hoc," in *Entre paradojas : a 50 años de los libros de texto gratuitos*, El Colegio de México, 2011, pp. 627-643.
- [2] M. Hoch and T. Dreyfus, "Recognising an algebraic structure," in *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education*, 2007, pp. 436-445.