YOU DO – YOU UNDERSTAND, YOU EXPLORE – YOU INVENT: THE FOURTH LEVEL OF INQUIRY-BASED LEARNING

Evgenia Sendova[1]

Abstract
This paper presents a personal history of the author's teaching experience going back to the roots of constructionism in Bulgaria and is supplemented by recent comments from her former students. A special kind of inquiry based learning – the open inquiry, is discussed based on the author's involvement in two institutions for doing scientific research at school age – the High School Institute of Mathematics and Informatics in Bulgaria and the Research Science Institute – in USA. Various approaches to preparing teachers for an Inquiry Based Math and Science Education are mentioned within the framework of recent European projects (MaSciL, Scientix, KeyCoMath).

Keywords inquiry-based learning, scientific research at school age, the art of mentoring in support of creativity, competences, RSI, HSSI

1. Introduction

1.1. What is still worth writing about?

When you hear "Constructionism" you think of creating something (artifacts or ideas) that could be shared. And what could it be that is still worth writing about after so many years the notion was coined by Seymour Papert and embodied in the Logo culture all over the world... A possible solution could be a paraphrase of André Gide's famous words: *Everything has been said before, but since nobody listens we have to keep repeating it!* But he also says: *What another would have said as well as you, do not say it; what another would have written as well, do not write it*. To find a good excuse to refer to the constructionist ideas I have been following all these years, and still find a new context that would be worth sharing with colleagues, I decided to supplement my memories by comments from former students of mine who have been doing research since school age. Such a feedback would bring something new for sure and hopefully contribute to educators' expectations associated with constructionism. Let us take a look back at the first seeds of constructionism in Bulgaria.

1.2. The roots of the Logo culture in Bulgaria through the eyes of international experts

It is thanks to the Logo culture [1] of sharing your joy of learning, of harnessing your mistakes in a creative context, of working with students and teachers as your partners in a research team, that I became involved in 1984 in the Research Group on Education (RGE) experiment [2-4] whose guiding principles were *learning by doing*, *guided discovery*, and *integrated school subjects*. My

[1] Institute of Mathematics and Informatics, Bulgarian Academy of Sciences

role was to participate in the development of educational materials – textbooks *Language and Mathematics* being an integration of mathematics, natural languages and Logo [5, 6], the *Mathematics and Informatics* bulletin for teachers, and of computer microworlds tuned to studying mathematics, languages, fine arts and music [7]. The exploratory activities in these microworlds made it possible for different talents to flourish and for the students and the teachers alike to become aware of their own creativity potential. The experimental RGE schools (2 in Sofia out of 29 total in the Bulgaria) were often visited by educators from abroad (Israel, Russia, UK, US). Below are my memories of several such visits.

In the early 80s, while visiting School #119 in Sofia a Russian educator and social philosopher (Simon Soloveychik) asked me to describe the RGE experiment in one sentence without using the word *computer.* I said: *Our children love going to school*. The immediate answer was: *Thank you, this is absolutely sufficient*. Indeed, my statement embodied the whole atmosphere of creativity of teachers and students alike, the computers being only part of the environment (but a natural means of expressing oneself in an integrated learning process). Of course, my spontaneous reaction might have been inspired by Paper's thoughts [8]: *Beyond the love for knowledge, is this principle: If you love what you learn, you'll get to love yourself more. And that has to be the goal of education, that each individual will come out with a sense of personal self-respect, empowerment, and love for oneself, because from that grow all the other loves: for people, for knowledge, for the society in which you live.*

Another great Russian educator and psychologist (Vasily Davydov) visited an RGE school in the town of Blagoevgrad, a day earlier than announced. When entering the school he smiled and said: *I know that you would show the best you have to a guest, but I want a realistic picture.* The teacher looked at me slightly surprised but I encouraged her to carry out her plan for the day – the lesson was exploring procedures for drawing spirals with parameters (for the angle of the turtle turn and the size of the initial segment). The self-similarity of the construction naturally led the 5th graders to writing a recursive procedure. By experimenting with various values for the ANGLE parameter the students found interesting patterns, they were able to create spirals with the shape of regular polygons and with a specific number of branches switching left or right. They modified the procedure by first introducing a parameter for the increment of SIZE and then by changing the rule of augmenting it (SIZE*2). Then the students decided to check what would happen if they fixed the SIZE and increased the ANGLE. The latter idea was born with the help of the teacher who was thus preparing the ground for experiments in science with processes depending on several parameters. Prof. Davydov was genuinely surprised admitting that he would suspect a preliminary setting hadn't he changed the date of his visit. Little did he know that such a creative atmosphere was typical for the *Language and Mathematics* classes.

Another visitor, Paul Goldenberg (from the Educational Development Center, USA), was invited in 1989 as an expert to evaluate the strategy of integrating informatics in the RGE curriculum. During his visit he worked with 4th and 5th graders (again from school #119). Ten years later, at a Eurologo conference in Sofia, he expressed the following views [9]:

> *Ideally, students should be able by high school to use the tools appropriately to solve non-routine problems that match their intellectual and mathematical development. Such an approach is strongly at odds with the too-common tendency to treat Logo as an elementary school toy, and then drop it altogether (or to replace it with another language, which is a bit like using a semester of Spanish as preparation for French). As a toy, any computer language is far too expensive in time to be worth the investment. (...) One's own natural language is best for conveying the semantics of a mathematical idea or situation; algebraic language is*

best at expressing and transforming quantitative or structural relationships; and computational language is optimal for describing processes and algorithms. That – especially the last two paragraphs – I feel like I learned in Sofia!

The visit of Seymour Papert (1987) in the Sofia School #2 offered various kinds of emotions. He could easily figure out the primitive Logo commands in Bulgarian and communicated in a friendly way with the 5th graders who worked on a Logo procedure for drawing a polygon. The teacher (Miroslava Bekyarova) had left the kids to work independently but there was a boy whose turtle *didn't obey* him. He looked at the teacher for help but neither she, nor I could see any error. So I took the gauntlet down to Papert. He enthusiastically rolled his sleeves and started trying out the commands in the command center – no effect. Finally he reduced the procedure's body to the simplest possible command – FORWARD 100. Again – no effect. Then he said in a triumphant voice: *A hardware problem!!!* At the end of the classes I encouraged the children to ask Papert anything they wanted. This was the first American they had met in person and I was ready for any question but the following: *Do you, in the US, have such a good teacher as Ms. Bekyarova?* You could guess the obvious answer. It is not surprising that Papert himself finished his talk at the international conference in Sofia that year by quoting this question [8]: *I was so moved, I didn't know what to say ... But I thought, "Isn't that wonderful?" There was something about the kind of work they were doing that made them feel this way about their teacher. Of course their teacher is a wonderful person, but we can create educational environments that bring out the love for the teacher and the love for everyone else there.*

1.3. Supporting teachers' creativity

Being aware of the importance of the teacher's role for any educational reform, the participants in the RGE informatics group transferred the positive experience of the early Logo days at university level and developed new university courses, in which the spirit of constructionism was in-built in a natural way. In 1989 the traditional core of mathematical disciplines taught at the Faculty of Mathematics and Informatics at Sofia University was enriched by a series of Logo based courses: *Logo programming, Working in a Lego-Logo environment, Informatics in the secondary school mathematics curriculum, Problem-oriented languages, Teaching mathematics in a laboratory type environment* [10]. In the case of the latter the teachers-to-be experienced the feeling of "doing mathematics" as opposed to the "correct" recipe-type activities. After years of studying very sophisticated mathematical facts they were put into situations where they could say: "Look at *my* construction!"; "Can you prove *my* theorem?" Furthermore, they learned that "not being afraid of being wrong" is a necessary condition for revealing one's creativity potential. Thus, we, the lecturers, expected that the spirit of discovery would be transferred to their pupils... Future teachers became convinced that studying mathematics is a mode of learning how to think and express oneself, how to behave socially. As a result they felt better prepared for the new role of the teacher – that of an advisor, stimulator, a participant into a creative process. Some of the in-service teachers also acted like researchers – they managed to create entire classes of mathematical problems that were new to the existing curriculum. Before proving their hypotheses they verified them with *Geomland* (a Logo-based *mathematical laboratory* [11] enabling constructions and experiments with Euclidean geometric objects) and investigated various extensions of the initial problems. Thus no longer did the pupils view their teacher as someone who was supposed to know "the best" solution to any problem in the textbook – for them their teacher became the author of new theorems and problems. One such teacher (Krassimir Dachev) shared with pride that a students of his (who had never seemed interested in anything but sports) had announced to his parents: "Today we learned a theorem discovered by our teacher!" And even more amazingly to them, he started explaining the very theorem [4].

Looking back at the challenges my trainee-teachers have overcome, I feel proud with their newly gained self-confidence, with their readiness to teach in an inquiry based style. My work with pre- and in-service teachers was inspired by the principle [12] *If we hope for a real positive change in education, we should bring today's and tomorrow's teachers in situations in which they would stop thinking about the future in terms of tests, exams or teaching pupils only. We should rather enable them experience what they are doing as intellectually exciting and joyful on its own right.* Such a spirit is part of our shared constructionism heritage and there are various manifestations of its being alive today. The inquiry based learning (IBL) is one of the approaches that are part of this heritage.

2. Inquiry based learning – a strategy preparing for the unpredictable

To keep up with the knowledge dynamics in a specific field, experts should acquire skills and competences such as: (i) inquiring and identifying relevant information; (ii) conducting their own explorations; (iii) applying creatively their findings; (iv) presenting the results in a way convincing for others [13]. An educational strategy towards this goal is based on the inquiry approach to learning. The recent developments in Bulgaria related to implementing and disseminating IBL are mainly in the framework of European Projects, e.g. *InnoMathEd, Fibonacci, MaSciL, KeyCoMath, Scientix* [14, 15]. These projects aim to develop and test (both in and outside of class) various learning environments which provide an appropriate platform for IBL. This led to the establishment of a *Virtual School Mathematics Laboratory* (VirMathLab) at IMI-BAS [16]. Hundreds of teachers in mathematics and information technologies have been included in seminars and workshops in the frames of the above projects and the most active of them have participated in the approbation and enrichment of these environments so as to tune them to the needs of their pupils To have the ability of approaching problems by different means, of looking at them from different angles requires time and efforts of different nature. When talking about the inquiry-based learning we should take into account that it is described as occurring at 4 main levels [17]: (i) *confirmation inquiry*; (ii) *structured inquiry*; (iii) *guided inquiry*, and (iv) *open inquiry*. Thus it is important to prepare teachers to work at all these levels. A crucial aspect of *constructionism* as an educational philosophy giving rise to the IBL is fostering situations in which the teacher has to join the students as an authentic co-learner. As Seymour Papert expresses it picturesquely in [1], *the best way to become a good carpenter is by participating with a good carpenter in the act of carpentering. By analogy, the way to become a good learner is by practicing with a good learner in an act of learning.* It is in the ideology behind the *VirMathLab* to enable the teachers to act as learners on their own and to share the act of learning with their pupils.

One of the RGE teachers (Neli Stoyanova) involved in *MaSciL*, *Scientix* and *KeyCoMath* shares:

> *When I decide to give problems appropriate for IBL, I don't think of the curriculum and the syllabus, I leave my students to inquire, to think, to combine, to create and to surpass me! I am glad when they feel like discoverers, when they decide to explore and find their own way, idea… In the past, they were used to asking Google first. But the IBL approach is very powerful; it motivates the students to be inquirers, to reveal their potential. I feel the constructionist's spirit reborn…*

Again, one of the big lessons of the constructionism is that *there can't be making without sense-making* [18]. And making sense of your inquiry requires sufficient time. Thus the real open inquiry learning requires additional educational forms.

3. Promoting scientific research at school age

Many researchers in gifted education express their belief that educational programs outside of schools are absolutely necessary for gifted children because they meet their special learning needs by providing more opportunities for independent inquiry, in-depth study, and accelerated learning [19]. In addition, such programs are a great chance to meet other bright kids who are fascinated by learning [20]. When working on research projects with secondary school students, the activities usually pass through several main phases [21]:

- Preparation phase – motivating the students for exploring a topic of interest by delivering short lectures, appropriate warm-up problems, assigning resources to study
- Research phase – engaging the students in research activities by formulating appropriate projects (short-term ones – up to 2 weeks) and long-term ones (from 6 weeks to 2 years), normally leading to a publication
- Presentation phase – building up skills for a written and oral presentation of the project
- *Passing on the torch* – teaching students to act like mentors

To put the Bulgarian students with special interests in math and science in a more realistic research situation, a *High School Institute in Mathematics and Informatics* (HSSI) was founded [22]. This institute inherited the good traditions of an earlier movement of the technically creative youth in Bulgaria and an international research program held in the USA – the *Research Science Institute* (RSI) [23] sponsored jointly by the Center for Excellence in Education (CEE), and the Massachusetts Institute of Technology (MIT).

3.1. The Research Science Institute

If I have to describe this program with one sentence it would be: "the place where to be extraordinary is the most ordinary thing"… This applies to the students, to the mentors, to the morning and evening lecturers and to all the rest officially and unofficially involved. Several words about RSI's founding and principles [24]:

The Research Science Institute (RSI) was developed by the Center for Excellence in Education (CEE), a non-profit educational foundation in McLean, Virginia. The Center was founded with the idea of nurturing young scholars to careers of excellence and leadership in science, mathematics, and technology. Central to CEE is the principle that talent in science and math fulfills its promise when it is nurtured from an early age. RSI is attended by approximately 80 high-school students from more than 25 nations. Once selected, the students come to MIT and work on a research project under the guidance of faculty, post-docs, and graduate students at MIT, Harvard, Boston University, and other research and industry institutions from the Boston-area. All the students chosen for the Institute have already acquired a deep interest and certain research experience in a scientific field of inquiry. Students work on their research projects for five weeks. At the conclusion of this internship, they write a paper summarizing their results and give an oral presentation of their work in front of a large audience at the RSI Symposium. Especially important in the process of preparation are the *milestones* – intermediate steps of the process. Typical milestones for the written presentation are: writing about a mini-project using the same sample as the one for the final paper; gradually filling the proposed sample starting with the background of the project, the literature studied and the methods used; considering partial cases and possible generalizations; classifying the cases of failure (in the case of mathematics projects), etc. Possible milestones for the oral presentation are: speaking for 3 min on a freely chosen topic, presenting the introductory part of the project for 5 min, etc. All the milestones are accompanied by a feedback from us, the tutors, who work closely with the students – we read and critique the draft papers, provide editorial remarks, suggest avenues of research and areas of additional background reading, give ideas for tuning the oral presentations to a specific audience, etc. In general, we are the *psychological oil* for the students' problems. To get an idea of the variety of topics of projects performed at RSI you might look at the compendiums of three consecutive years [25] containing the abstracts of all the written reports with five selected as representative which are published in full.

3.2. An example of a Logo-related project within RSI

Bartosz Tarnawski (RSI'12) arrived at RSI with a solid background in Computer Science, mathematics, physics and biology. His project (mentored by Alec Resnick, RSI'03) was an extension of Logo language towards constructionist education in mathematics (which he called *Transformland).* Bartosz started with reading *Turtle Geometry* [26] and found the ideas in the book *really moving*, e.g. the idea that any surface may be obtained by appropriate gluing of small polygons. Here is the abstract of his paper published in the compendium of RSI'12:

> *The paper deals with the design of an extension of the Logo programming language. The extension consists of two environments, each of which is specific to one mathematical domain transformation of the plane- and differential geometry. In the first environment, called "Transformland"* [Fig. 1], *there are two viewports, one of them displaying the original objects and the other showing the same objects under angle-preserving transformations. In the second environment the turtle's world is a curved or a piecewise at surface. The world is shown from the turtle's point of view – locally it resembles the plane. This effect is obtained by an orthographic projection of the surface. This environment is designed in support of intuitive learning of differential geometry.*

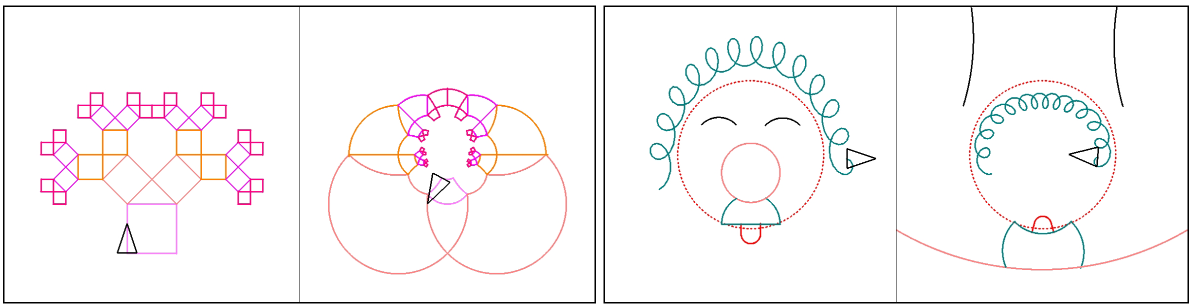

Fig. 1 The Pythagoras tree and a "simplified" Einstein under inversion in *Transformland*

Recently I asked Bartosz to look back at his RSI experience:

> *At the beginning of RSI my mentor Alek Resnick suggested developing the project in a Lisp-derived language Clojure. This was quite a big surprise to me – I felt comfortable only with C++ and thought that all sensible programs were written in similar (imperative) languages. Due to the subject of my project I had to recall Logo. Some of the solutions in the syntax of the language had always seemed quite weird to me. For example in C++ I used arrays all the time and lack of them seemed to be a major drawback. However, during the project I began to realize that the structure of a programming language is closely related to the domains it is meant to be used in. Hence C++ may be good for programming contests, where little*

*execution time and short code length are most important, whereas Logo is way better for education-, art-, language-, ...-, related tasks. Experimenting with new programming languages seems to be a lot of fun. One of the crucial things is to understand at the very beginning **what** the language was designed for and **how** it should be used.*

This observation is very mature and gets at one of the problems with informatics education in my country, viz. that the students with high interests and potential in informatics are mainly trained for competitions. Thus one of the main ideas behind establishing the High School Students Institute (HSSI) in Bulgaria was to implement RSI-like activities in mathematics and informatics context, taking into account the local conditions and traditions.

3.3. HSSI in Bulgaria

As one of the founders of the Bulgarian school for identifying and developing mathematically gifted students, Petar Kenderov, often says [26]: *Talent is a resource which, unlike the ores, could vanish if not discovered early enough…* With this idea in mind and following the inspiring example of RSI, in the year 2000 a *High School Institute in Mathematics and Informatics* (HSSI) was founded by the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences, the Union of Bulgarian Mathematicians, St. Cyril and St. Methodius International Foundation, and Evrika Foundation in response to the decision of UNESCO to declare that year as "World Year of Mathematics". The activities of HSSI are focused on projects in mathematics, informatics and information technologies [24]. The participants in HSSI are high school students between 8th and 12th grade, mainly from specialized Science and Mathematics Secondary Schools in the country. Every participant in HSSI works on a freely chosen topic in mathematics, informatics and/or IT under the guidance of a teacher or another specialist. A written presentation of the project is sent to HSSI. All papers are reviewed by specialists. The best research projects are accepted for a presentation at the conference sessions of HSSI.

The High School Students' Conference is usually attended by more than 200 students, teachers, researchers, journalists. Based on the merits of the paper and presentation, the Jury judges the works and selects the best ones. Their authors are invited to take part in an interview for selecting two Bulgarian participants in RSI and to participate in the School Section of the Annual Conference of the Union of the Bulgarian Mathematicians. The authors of the best projects from the School Section are invited to participate in a three-week Research Summer School. During the first two weeks, lectures and practical courses in mathematics and informatics are delivered by specialists from universities, academic institutions and software companies. The main goal of the training is to extend the students' knowledge in topics related to their interests and to offer new problems, potentially the core of short-term projects. During the third week the participants present the results of their short-term projects and exchange ideas for further studies. To help teachers improve their mentoring skills, a High School Teachers Workshop is organized during the third week of the Research Summer School. Participants are the research advisors of the students' projects, presented at the events of HSSI during the school year.

4. To do research vs to be a mentor

In a recent improvised interview I asked some of my former students, alumni from HSSI and RSI, about the influence these institutions have had on their further development and on the difference between doing and mentoring scientific research. Here are some of their answers:

Yanitsa Pehova – (HSSI, currently undergraduate in Cambridge University, UK)

The HSSI is all very grown-up: you work with a supervisor on a research project and then present what you've done by submitting a paper for a conference. And yes, by "research project" I mean actual research. It's obvious that no 16-year-old could work on weak- operator topology but you'll be surprised how much a smart and determined teenager can achieve with supervision. With a passion for mathematics, I naturally went to several HSSI summer schools and conferences. Looking back at the work I did and the friends I made, it's safe to say that HSSI cultivated me as a mathematician and raised me as a human being. It did its job pretty well too, what came in was a 15-year-old girl and what came out was a Cambridge offer-holder… Being a supervisor for HSSI has been one of the most rewarding things I have ever done. As a mathematician, I was more or less raised by the conferences and summer schools during which HSSI set me on an academic track and this is something I'm surely grateful for. So as a supervisor, I both got to experience the atmosphere of these conferences one more time, and had the opportunity to express my gratitude towards the people who helped me keep my interest in mathematics alive… People keep saying the younger generation is less this and less that, and sometimes we have no choice but to believe them, but the experience I had working with my student, and the other projects I saw at the conference, have surely lifted my hopes up for the future of mathematics.*

Katerina Velcheva (HSSI and RSI'10, currently student and TA at MIT)

My RSI project (The competitiveness of Binned Free List on-line algorithm with an optimal off-line algorithm for heap-storage allocation) was more mathematical than CS. From my entire experience in doing research I think that what matters the most is not to give up and not to lose hope! Sometimes the results come slowly but it is worth pursuing them. There is no greater euphoria than proving something you have been struggling with for months. Sometimes it is better to leave the problem for a day or two (even for a semester) and go back to it from a new perspective… To be a mentor is an activity in which at the beginning one expects to give only and not to receive anything in return. The truth is that there is no greater reward than seeing the euphoria of your mentees when they learn and discover something new or solve a problem which has taken them a significant amount of time. As someone who has been a passionate competitor in mathematics and informatics as well as author of research projects I thought that the greatest joy is to solve a challenging problem. Today, while guiding two high school students from my native town I understand I have been wrong – to witness their happiness after they have achieved reportable results, makes me even more proud... Last, but not least, is the gratitude I get – to hear someone of my MIT freshmen exclaiming: "O-o-o-h, this is so cool" after I have found an elegant proof or to read in their evaluations: "She loves math so much, that she makes me love it too..." is a great feeling which gives me energy to finish my own homework (which might take me the whole night).

Kalina Petrova (HSSI, RSI'12, student at Princeton)

HSSI has played a great role in my learning how to write scientific text, how to present my achievements and how to speak in public. The conferences and the summer schools have given me the kind of scientific interest that drives me to explore every unfamiliar concept and to delve into every unconventional idea I get. In 2011 I took part in programming a driving simulator designed to help future and present drivers learn the rules of the road and enhance their understanding of the dynamics of driving. My team presented the simulator at a conference in Bulgaria and after that we took part with it in the European Union Contest for Young Scientists. This was the first big project I have ever taken part in and as such it changed my perspective on programming immensely… Working on a real piece of software at

RSI definitely had its challenges and it helped me develop my thinking a lot… What I did not like about programming in the context of a big project was that it required dealing with a lot of time consuming details, not particularly insightful on their own. For a person like me who is mostly interested in the ideological platform that is used for achieving a result (actually, sometimes we just do theory for the sake of theory itself), taking care of details can kill the magic of it all… In my high school years I had the opportunity to (try to) conduct scientific research, both on my own and with the help of others. There are many people that have contributed to the person I am right now and to the fact that I managed to get admitted to Princeton University. I would like to help other people from my country develop their full potential and make the most of their high school experience. For this reason for the past two years I have been mentoring a girl who goes to the high school I went to, helping her with the research she conducts for the HSSI events. It has been extremely fulfilling watching her master her analytical thinking, her public speaking skills and her ability to write a good scientific paper. I intend to continue helping talented students with whatever I can.

Konstantin Delchev (HSSI, Director of Intel ISEF affiliated fair)

What did HSSI give me? Basically it taught me how to do research in the fields of mathematics and informatics. The change from the schoolboy mentality, where you have to solve the problems on the blackboard (or else) to the scientist mentality, where you have a problem that you investigate and no one knows what will happen. To a large extent, this is what pushed me towards a scientific career. What is my motivation to work with students? A lot was invested into me, in terms of time and efforts from people who were capable and experienced scientists in their respective fields. They had decided to detract time from their own research to work with us. After I graduated, I decided I should return the favor and in turn start helping students. I still feel I have much to learn from the ways I was taught, but the success of my own students indicates that I might be doing some things right, after all... The hardest part of working with students is actually the beginning – the research topic. Too challenging research topic means no results and tons of frustration. A low hanging fruit means no meaningful results and nothing learned. And it is not like you can always figure out the difficulty of an open problem in advance.

3. Conclusion

In a lecture for RSI'13 students Dudley R. Herschbach, a Nobel Laureate in chemistry, shared with the students that *teaching is very valuable to your research.* This thought is well supported by the sharings of my students quoted above. By mentoring and guiding the next generations of young minds they also pass the awareness that *approaching the work with genuine interest in research and discovery rather than winning competitions is far more valuable in the long run* [27].

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